THE STRESSEDLY-DEFORMED STATE OF A RECTANGULAR PLATE WITH RECTANGULAR CUT

НАПРЯЖЕНННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ ПРЯМОУГОЛЬНОЙ ПЛАСТИНЫ С ПРЯМОУГОЛЬНЫМ РАЗРЕЗОМ

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Abstract: *A new effective calculation method for the stability of a rectangular plate with rectangular cut is given. The solutions of a new class of problems concerning plates with violations of regularity in the conditions of physically and geometrically nonlinear deformations are obtained. These solutions are in accordance with the special features of stressedly-deformed state in the zones of stress concentration and lead to the simple algorithms of calculation.*
***KEYWORDS**: PLATES, STABILITY, CUTS, HOLES, EQUATIONS, CALCULATION, DISCONTINUOUS PARAMETERS*

1. *Introduction*

Construction of long-span facilities and application of contemporary low-modulus materials with high strength characteristics in practice leads to the necessity of taking into account large saggings in comparison with thickness during calculation of thin-walled constructions.

The development of different branches of industry and building is connected with improvement of existing and creation of new thin-walled constructions, which consist of shells, plates, rods, having reinforcements, holes, cuts, point supports. The group of mentioned irregularities is conventionally called as discontinuous parameters.

In the present work the methods of taking into account cuts and rectangular holes in a longitudinally compressed plate, developed earlier in the works of B. K. Mikhailov and his followers [1, 2, 3, 4], are extended by us for the problem of estimation of the deformity and stability of a plate with rectangular cut.

As the basis of solutions it is assumed the method of discontinuous functions, consisting in the fact that a desired solution is represented in the form of linear combinations of their regular and special discontinuous functions with unknown coefficients.

2. *Solution of considering problems*

Let us consider a rectangular plate with cuts *a*×*b*, having M cuts, parallel to side *b*. Orienting the coordinate axes, as it is shown in figure (Fig. 1), due to [1], let us present the function of sagging and the angle of rotation as follows

$$W^* = W + \sum_{i=1}^{m} \Delta W_i 1_{ixi} 1_{iyy} + \sum_{j=1}^{n} \Delta W_j 1_{jy} 1_{jxx},$$

$$\gamma_1^* = \gamma + \sum_{i=1}^{m} \Delta\gamma_{1i} 1_{ixiyy} + \sum_{j=1}^{n} \Delta\gamma_{1j} 1_{jy} 1_{xx} +$$

$$(1) \quad + \sum_{i=1}^{m} \Delta W_i \delta_{ix} 1_{iyy},$$

$$\gamma_2^* = \gamma_2 + \sum_{i=1}^{m} \Delta\gamma_{2i} 1_{ixiyy} + \sum_{j=1}^{n} \Delta\gamma_{2j} 1_{jy} 1_{jxx} +$$

$$+ \sum_{i=1}^{m} \Delta W_j \delta_{iy} 1_{jxx},$$

where $1_{ix} = 1(x - x_i)$, $1_{jy} = 1(y - y_j)$ are unit functions; $-1(x - x_{2j})$ – stepped functions, made up of unit functions.

$\delta_{ix} = \delta(x - x_i)$, $\delta_{iy} = \delta(y - y_j)$ – delta-functions, ΔW_i, ΔW_j – the mutual displacements of points on the edges of cuts; Δj_{1i} Δj_{2i}, $\Delta\gamma_{1j}$, $\Delta\gamma_{2j}$ – the angles of fractures and mutual rotations of the edges of cuts.

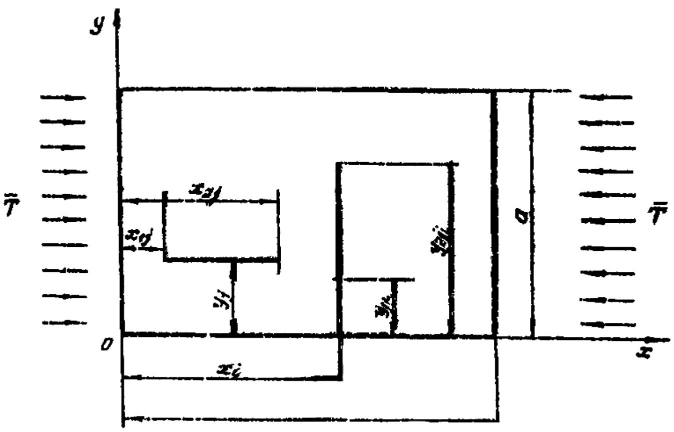

Fig. 1. Rectangular plate with cuts *a*×*b*, having M cuts, parallel to side *b*

For representation of the basic idea of solution let us consider the case, when a plate with one cut is compressed by load *T*, applied in the direction of axis *X* and distributed along sides $x = 0$, $x = a$.

Then, after substitution of expressions (1), and using the relationships of elasticity

$$M_i = D_1\left(W_x^{*''} + \mu W_y^{*''}\right);\ M_y = D_2\left(W_y^{*''} + \mu W_x^{*''}\right);$$

(2) $$H = D_3 \frac{1\cdot\mu}{2}\left(y_{xy}^{*1} + y_{2x}^{*1}\right),$$

into equilibrium equation with respect to the bending moments

(3) $$\frac{\partial^2 M_1^*}{\partial x^2} + 2\frac{\partial^2 H^*}{\partial x \partial y} + \frac{\partial^2 M_2^*}{\partial y^2} = \frac{P}{D}$$

and representing the component of external load in the form

(4) $$P = -T\frac{\partial^2 W^k}{\partial x^2}$$

we will obtain the equation, which corresponds to non-equilibrium, i. e. to the critical state

$$D_1\frac{\partial^4 W}{\partial x^4} + 2D_3\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 W}{\partial y^4} = -T(W_x'' +$$
$$+\Delta W\delta_x' 1_{yy} + \Delta\gamma_1\delta_x 1_{yy}) - [D_1\Delta W\delta_x'' + D_3 W_y''\delta_x' +$$
$$+D_2 W_y^{IV} 1_x]1_{yy} - [D_1\Delta\gamma_1\delta_x'' + D_3\Delta\gamma_{1y}''\delta_x]1_{yy} -$$

(5) $$-D_2\cdot 2\Delta W'''\delta_{yy}1_x - D_2\Delta W''\delta_{yy}' 1_x - D_3\Delta\gamma_{1y}\delta_x'\delta_{yy},$$

where $D_3 = D_1 + 2D_{xy}$.

In the problem of stability analysis it is possible to take $\Delta W = 0$, since the presence only of fracture, even without the divergence of edges, already causes loss of stability. Then equation (5) can be written in the form

$$D_1\frac{\partial^4 W}{\partial x^4} + 2D_3\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 W}{\partial y^4} = -T(W_x'' +$$
$$+\Delta\gamma\delta_x 1_{yy}) - (D_1\Delta\gamma_1\delta_x'' + D_3\Delta\gamma_{1y}''\delta_x)1_{yy} +$$

(6) $$+D_3\gamma_{1y}'\delta_x\delta_{yy}.$$

From this equation it is possible to find critical load T by assigning sagging W and angle of fracture $\Delta\gamma$ with the system of approximating functions, corresponding to the boundary conditions on contour, and then to use one of variation methods. This method gives completely acceptable results in the case when the plate is continuous and the series for functions W and γ_1 converge sufficiently rapidly. However, in the case of fracture the sagging is a discontinuous function and the convergence of series in approximation by regular functions sharply worsens. The numerical analysis of some analogous problems for an isotropic plate shows that for achievement of the necessary precision (error 5%) there should be held 10 or 11 terms of the series.

For obtaining sufficiently rapid convergence in the present work we use another method, based on the solution of corresponding static problem and which allows representing the function of sagging as an expansion by discontinuous functions. The essence of this method lies in the fact that since summand $-T(W_x'' + \Delta\gamma_1\delta_x 1_{yy})$ in the right side of equation (6) has a dimensionality of load, it can be considered as a certain given external load, then at first it is being solved the problem on plate bending under action of concentrated force, applied at point with coordinates x_1, y_1. Then the solution of the corresponding equation

$$\times\delta(y - y_1) - (D_1\Delta\gamma_1\delta_x'' + D_3\Delta\gamma_{1y}''\delta_x)1_{yy} -$$

(7) $$-D_3\Delta\gamma_{1y}'\delta_x'\delta_{yy}.$$

will have the form [2].

(8) $$W_{kx} = \sum (W_{ok} + \Delta\gamma_k f_k)\sin\beta xy,$$

where W_{ok} is a solution of the equation.

With the expansion in series in terms of x coordinate it will take the form

(9) $$\left(D_1\frac{d^4}{dx^4} - 2D_3\beta_k^2\frac{\partial^2}{dx^2} + D_2\beta_k^4\right)W_{ok} = A_k\delta(x - x_1).$$

With the expansion in series in terms of x coordinate it will take the form

(10) $$W_{obx} = \frac{\frac{4P}{ab}\sum_{\ell}^{x}\sin d_e x_1 \sin\beta_k y_1}{(D_1\alpha_e^4 + 2D_3\alpha_e^2\beta_k^2 + D_2\beta_k^4)}\sin\alpha_e x.$$

Coefficient $\Delta\gamma_k$ will be found from the condition of equality to zero of moment M_1 on the line of cut. From this condition with $k = \ell = 1$ we obtain

$$\Delta\gamma_k = \frac{D_1 W_{ok}'' - \mu D_2\beta_k^2 W_{ok}}{D_1 f_k'' - \mu D_2\beta_k^2 f},\ x = x_1.$$

Function f_x is determined by the expression [2]

$$f_k = -(D_3\psi_{kx} + D_3\beta_k^2\psi_{kx})\alpha_1 - D_3 d_2\beta_k\psi_{kx}',$$

where ψ_{kx} is a solution of the equation of the form

$$D_1\frac{d^4\psi_{kx}}{dx^4} - 2D_3\beta_k^2\frac{d^2\psi_{kx}}{dx^2} + D_2\psi_{kx} = \delta(x - x_1).$$

Then expression (8) is represented as follows:

$$W = \sum W_{ok}(1 + \bar{\gamma}_1 f_k)\sin\beta_k y,$$

where $$\Delta\bar{\gamma} = \frac{D_1\alpha_1^2 + \mu D_2\beta_1^2}{D_1 f_1'' - \mu D_2\beta_1^2 f_1}.$$

The condition of critical state can be obtained by considering force R in expression (10) as the load distributed over the area of infinitely small element dx_1, dy_1. Then, by replacing it with equivalent load $TW_x'' dx_1, dy_1$ in the deformed surface and integrating throughout the entire area of the plate with respect to coordinates $x_1 y_1$, we will obtain the formula for determination of the critical load, which takes the form

$$T_{kp} = \frac{D_1 \alpha_e^4 + 2D_3 \alpha_1^2 \beta_1^2 + D_2 \beta_1}{\alpha_1^2 (1 + \Delta \bar{\gamma} \bar{\bar{f}}_k)} . \tag{11}$$

3. *Conclusion*

It is received and investigated the version of differential equations, which allows investigating the stress-strained state of large class of shells and lamellar systems with edges, cuts and holes under conditions of nonlinear deformation on a unified base. Different simplified versions of these equations are received. The used theoretical model reflects the elements of structures and constructions, carriers and fillers, used in aviashipbuilding, chemical instrument manufacture and other areas of technology.

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