

# ESTIMATION OF INSTANTANEOUS CENTER OF ROTATION FROM OBSERVED MOTION

## ОЦЕНКА МГНОВЕННОГО ЦЕНТРА ВРАЩЕНИЯ В ЗАВИСИМОСТИ НАБЛЮДАЕМОГО ДВИЖЕНИЯ

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**Abstract:** The aim of the paper is to describe numerically the motion of a moving rigid object which is observed using filming equipment. Only three arbitrary points are used as markers on the object and the challenge is to recreate the velocity field of the object from measured velocities in these three points. The methodology used dwells on the fact that any general motion can be in any instant described as pure rotational motion around instantaneous center of rotation. Thus the main goal is to design a straightforward procedure for the estimation of instantaneous center of rotation and corresponding angular velocity accounting for possible errors in known velocities arisen from reading of marker points coordinates in each digitalized film frame.

**KEYWORDS:** INSTANTANEOUS CENTER OF ROTATION, VELOCITY FIELD, OBSERVED MOTION

### 1. Introduction

The research on the subject portrayed in the abstract was initiated by the need of simple and cheap methodology for obtaining the general motion of any object in the form of computer data with possibility of further analysis of the data in order to obtain basic properties of given object subjected to motion as for example center of gravity and/or suspension parameters in the case of vehicles. There are other methods than the one presented here which are very simple [1, 2] but the results as applied to the problem were not sufficient. Using rotation matrix for transformation including displacements is very tricky as the center of rotation and displacements are interrelated. On top of that the angle of rotation is passed as an argument in trigonometric functions and if working with errors and uncertainties, one can come with four different angles or even cosine function larger than 1 in the rotation transformation matrix. For the sensitivity of this work this was unacceptable and so an optimization method was utilized to reduce the errors in measured velocities to minimum and at the same time finding the instantaneous center of rotation and angular velocity. There is no way to include unknown displacements or translational velocities as the center of gravity is also unknown. The only possibility is to transform the general movement of the body in any instant to pure rotational movement and thus reducing the number of unknown quantities. Nevertheless it is further possible to analyze the data, recover center of gravity and also translational components of the velocities and thus reconstruct the general motion as a superposition of translation and rotation around the center of gravity.

### 2. Measurements and methodology

For the measurement an unmarked AKTIS class vehicle subjected to random excitations was used. The motion of the vehicle was recorded from the side using camera with PAL image quality. This as it turned out was not so lucky decision but served the purpose as the worst case scenario was used. For better accuracy though it is recommended to use HD quality camera on tripod and marker points on the vehicle in the form of colored crosses. Part of the vehicle with marker points added is shown in fig. 1. The velocities in the three points were estimated on per frame basis simply by comparing two adjacent frames of footage and marking some significant spots on the vehicle. Thus the velocity obtained is in pixels per frame. The ideology is depicted in fig. 2. Note that by zooming the selection of the frame the picture gets pixelated which makes the estimation of exact coordinates impossible and errors should be incorporated into calculations. In this case one pixel in the frame is approximately 7.4 mm in reality and time between two adjacent frames is approximately 0.04 s. One can then convert the velocities to be in mm/s. But this is hardly necessary to do apriori

and can be done after all calculations have been performed by applying the conversion on the velocity field.

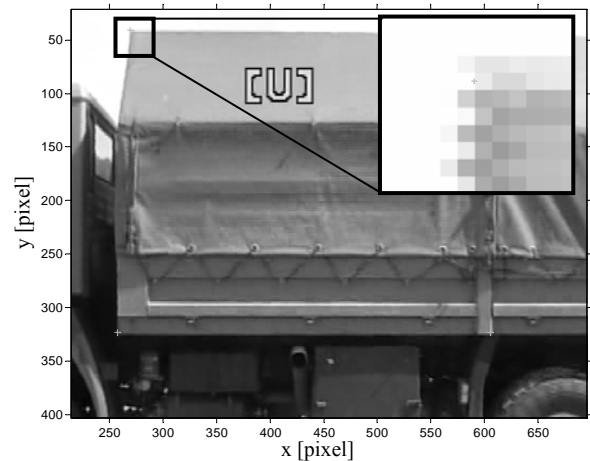


Fig. 1 Vehicle with marker points (+) and zoom to one of them

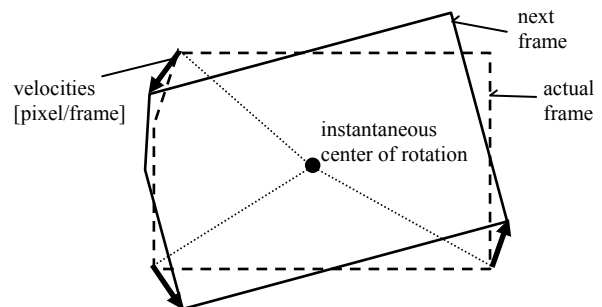


Fig. 2 Estimating the velocities and relationship to center of instantaneous rotation

All velocities of the rigid object are perpendicular to the line connecting point in which the velocity originates with the instantaneous center of rotation. The magnitude of velocity is linearly dependent on the distance from the center of rotation and on the angular velocity. Thus the mathematical expression can be formulated as follows

$$(1) \quad v = \omega \times r$$

where  $v$  is velocity vector,  $\omega$  is angular velocity vector and  $r$  is position vector. The only known quantity here is the left hand side

velocity, in our case in three points. Equation (1) can further be written in the component form as follows

$$(2) \quad \begin{aligned} v_x &= -\omega r_y = -\omega(x_p - x) \\ v_y &= \omega r_x = \omega(y_p - y) \end{aligned}$$

where  $v_x$  and  $v_y$  are components of velocity vector acting in point with coordinates  $x$  and  $y$ , and  $x_p$ ,  $y_p$  are coordinates of the instantaneous center of rotation. The unknown quantities here are angular velocity  $\omega$  and  $x_p$ ,  $y_p$  coordinates. Note that these three quantities are multiplied in the equation and therefore there is no way to assemble a system of linear equations. It would be possible if the measured velocities were exact to first determine the angular velocity and afterwards compute coordinates of the instantaneous center of rotation. But this is not the case as the lines perpendicular to the uncertain velocities would not even intersect in the same point (fig. 3). Thus the only way is to formulate the problem as follows

$$(3) \quad f(x_p, y_p, \omega) = |v - \omega \times r| = MIN.$$

This is an optimization problem where the minimum deviation from measured velocities is quantified and adequately searching for using three unknown variables as mention above. The function which to minimize is shown in fig. 4 for one particular angular velocity. So even though it is three dimensional picture one can view it as only one slice of four dimensional reality for given angular velocity. One needs to realize that the minimum is a function of three variables that means also a function of  $\omega$ .

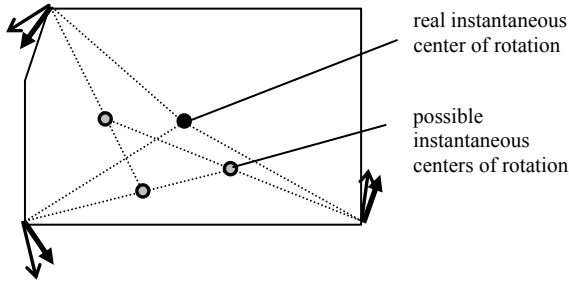


Fig. 3 Variation in velocity and the implication upon instantaneous center of rotation

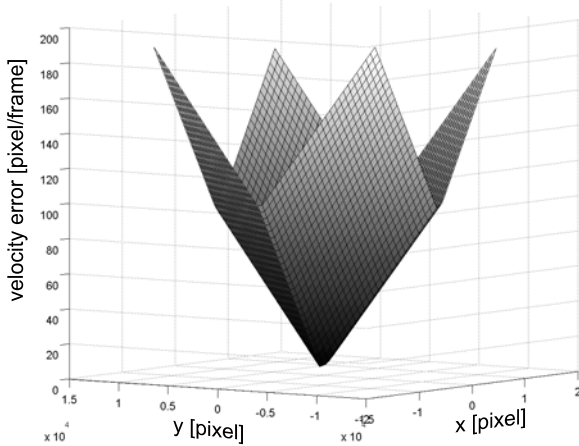


Fig. 4 Function to be minimized dependent on  $x_p$  and  $y_p$

The optimization algorithm is augmented steepest descent algorithm with added variable step size to the procedure to

adaptively refine the step when nearing the minimum. The derivatives of the function (3) are obtained through the central difference method. The core of the algorithm is as follows

Algorithm 1:

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initial guess for xp, yp, ω
do for number of iterations

$$x_p - = \frac{f(x_p + dx_p, y_p, \omega) - f(x_p - dx_p, y_p, \omega)}{2dx_p} dt$$


$$y_p - = \frac{f(x_p, y_p + dy_p, \omega) - f(x_p, y_p - dy_p, \omega)}{2dy_p} dt$$


$$\omega - = \frac{f(x_p, y_p, \omega + d\omega) - f(x_p, y_p, \omega - d\omega)}{2d\omega} dt$$


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where  $dx_p$ ,  $dy_p$  and  $d\omega$  are small differences in corresponding directions which all were set to  $1e-6$  for this purpose. The right hand side of the equations is actually the gradient of the function in corresponding direction which is multiplied by step augmenter  $dt$  to get the final step size by which the variables are advanced. After the procedure has converged to a minimum value of the function  $f$ , the optimal coordinates of instantaneous center of rotation and optimal angular velocity are stored. This can be repeated for each frame. The uncertainty in the measured velocities is accounted for by adding/substracting some small value representing possible error.

$$(4) \quad v = v \pm v_{err}$$

In the sense of Monte Carlo simulation, the above algorithm is performed for each velocity variation  $v_{err}$  to obtain variation in instantaneous rotation center coordinates and angular velocity, thus to estimate rate of sensitivity to imperfect input parameters. The velocity in any point can be calculated using the equation (1) except now the velocity is unknown and the angular velocity and position vector are known as obtained by the optimization procedure.

### 3. Results

The results here will be shown for restoration of velocities for one frame and the whole procedure can be automatized for any number of frames from the footage. Resolution used to capture the footage was set to standard PAL resolution which is  $720 \times 576$  pixels. Number of frames per second was set to the standard 25 frames/s. Three points were identified on the vehicle and their position was tracked in each frame. The velocities were then estimated as a difference in coordinates in two consecutive frames. Coordinates and estimated velocities for the frame of interest are shown in tab.1.

Table 1:

Point no.	Coordinates [pixels]		Velocity [pixel/frame]	
	x	y	$v_x$	$v_y$
1	270	42	-3.8	1.7
2	257	325	-3.0	2.0
3	605	325	-2.5	-0.4

In order to find the instantaneous center of rotation the now known velocities in these three points have to be optimized so the lines perpendicular to them intersect in one point and the error between estimated and optimized velocities is minimal. It is advisable to use even higher number of points as the error is significantly reduced this way. Even though the function (3) is minimized, the error

between the velocities is not known prior to the procedure and thus we do not know the minimal value of the function which represents the velocity error. The termination of the procedure is not appropriate based on some minimal delta value which the minimum should be less or equal to. So the decision fell to terminate the procedure after prescribed number of iterations. It is also possible to end the procedure when the function is changing only slightly or when it is oscillating around some value with small enough amplitude. As it is shown in fig.5 and fig.6 the number of iterations, in this case set to 6000, worked pretty well and the function and also the angular velocity stabilized themselves after the minimum was achieved. The velocity error is calculated as a sum of absolute errors in all three velocities and the final value reads 1.1934 pixel/frame. Divided among three velocities the average error in each velocity is 0.3978 pixel/frame.

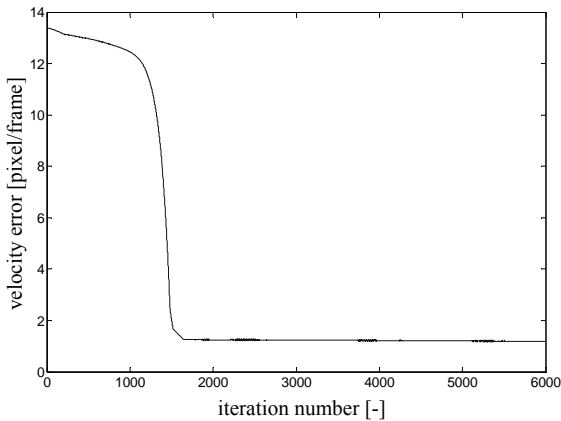


Fig. 5 Process of the minimization of the function

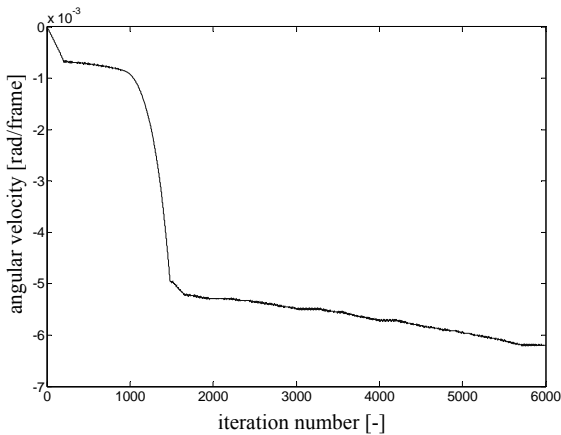


Fig. 6 Angular velocity value during the minimization

The estimated velocities and the optimized ones together with difference/error between them recalculated to mm/s are shown in tab. 2.

Table 2:

Point no.	Estimated velocity [mm/s]		Optimized velocity [mm/s]		Error [mm/s]	
	$V_x$	$V_y$	$V_x$	$V_y$	$V_x$	$V_y$
1	-703.0	314.5	-778.5	314.5	-75.5	0.0
2	-555.0	370.0	-455.9	325.5	99.1	-44.5
3	-462.5	-74.0	-461.9	-73.8	0.5	0.2

The errors in the table seems to be enormous, but one has to realize that this is all due to pixelation of the camera picture and thus error in estimates equalling 1 pixel/frame is in reality 185 mm/s. So the maximal error is 'only' approximately 0.5 pixel/frame.

The initial guess for the optimization was set to  $x_p = 0$  pixels,  $y_p = 300$  pixels and  $\omega = 0$  rad/frame. The coordinates of the instantaneous center of rotation after the optimization are  $x_p = 534$  pixels,  $y_p = 719.6$  pixels and the angular velocity is  $\omega = -0.062$  rad/frame. The instantaneous center of rotation converges to its optimal position along the steepest gradient of the minimization function. The path during the iteration process is shown in fig. 6 together with the estimated velocities and optimized velocities which are almost blending into each other.

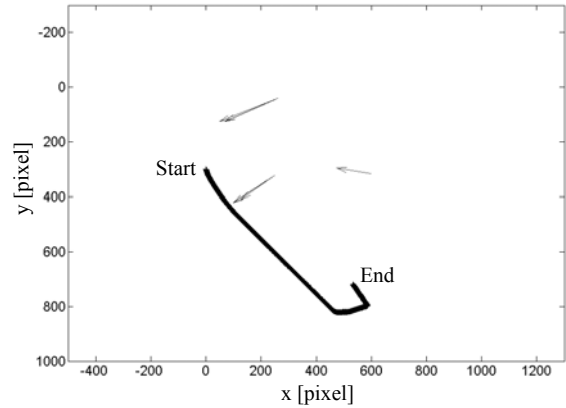


Fig. 7 Path of the instantaneous center of rotation during optimization

It was mentioned above that the errors in velocities are related to the pixelation of the processed image and thus it is interesting how the deviations in velocities affect the position of the instantaneous center of rotation. The sensitivity analysis was performed for two cases of velocity deviations. To each velocity component for each velocity was added variation of certain degree. In the first case the variation ranged from -0.5 to 0.5 pixel/frame and in the second case the range is from -0.2 to 0.2 pixel/frame. The deviation in position of the instantaneous center of rotation is shown in fig. 8. It can be seen that this problem is highly nonlinear and thus the position is varying significantly upon small changes in velocity vectors. The analysis was performed in the monte carlo sense with random variations for 700 iterations. The maximum deviation for  $x_p$  and  $y_p$  is shown in tab. 3.

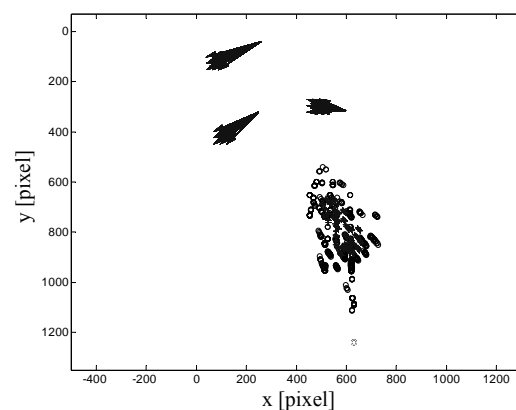


Fig. 8 Sensitivity analysis

Table 3:

Velocity deviation [pixel/frame]	$x_p$ range [pixel]	$y_p$ range [pixel]
-0.5 .. 0.5	334	635
-0.2 .. 0.2	121	210

After the coordinates and angular velocity have been established it is possible to restore the velocity field on the vehicle. The

velocities components are varying linearly along the axes. In this case it would be possible to perform linear regression to re-establish the velocity field but this approach was shown as inappropriate for the minimization of error in the velocity magnitude. Thus the optimization procedure performed much better. The velocity field for one frame is shown in fig. 9.

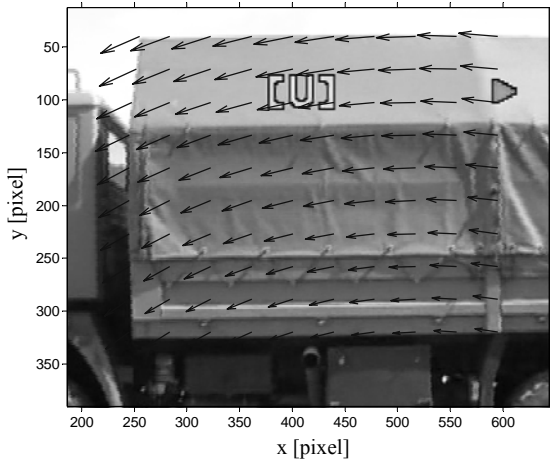


Fig. 9 Velocity field on the vehicle

The velocity represents a cone in 3D representation with the top representing the instantaneous rotational center. Thus the magnitude of velocity along the lower edge of the car trunk is a representation of a cut through the cone with plane parallel to the axis of the cone and this is a parabolic relation shown in fig 10.

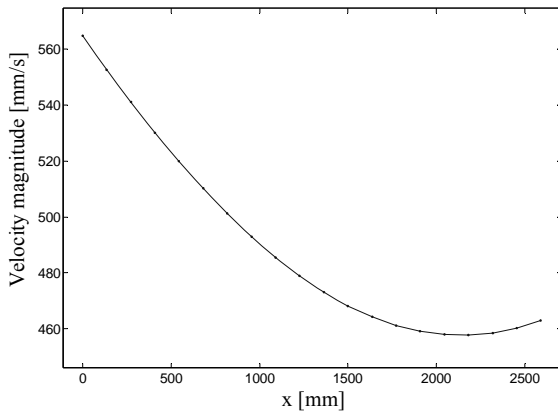


Fig. 10 Velocity magnitude along the lower edge of the velocity field

#### 4. Conclusions

In the paper is shown procedure to retrieve instantaneous center of rotation and velocity field from observed motion. An iteration method was utilised to find an optimal velocities from velocities estimators in marker points on a vehicle. Along with the condition of minimal error in velocities, the optimal position of instantaneous center of rotation was retrieved and also angular velocity. It is shown high nonlinearity in the problem using sensitivity analysis and deviations in velocities. The higher errors are due to low cost equipment used thus the pixelation took its toll. It is highly recommended to use high quality equipment capable of high definition resolution. The other alternative is to use equipment for vibration analysis and using several sensors it would be possible to evaluate the velocities in the points with sensors attached with vastly improved accuracy. The procedure was shown for one frame in this paper but extension to more frames is straightforward and can be automatized. The digital motion can be further processed and obtain center of gravity and/or suspension parameters from the motion as an example but there is certainly more applications.

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#### References

- [1] Collomosse, J. P., and P. M. Hall. Video Motion Analysis for the Synthesis of Dynamic Cues and Futurist Art. In *Graph. Models* 68(5), 2006, p. 402-414.
- [2] Taycher, L., J.W. Fisher III, T. Darrell. Recovering articulated model topology from observed rigid motion. In *proc. NIPS*, December 9-12, 2002, p. 1311–1318.