NEW THEORY OF ROTOR DYNAMICS: DYNAMICS OF OUTBOARD ROTOR WITH QUASI-STATIC UNBALANCE AT SUPERCRITICAL VELOCITIES

НОВАЯ ТЕОРИЯ ДИНАМИКИ РОТОРА: ДИНАМИКА КОНСОЛЬНОГО РОТОРА С КВАЗИСТАТИЧЕСКОЙ НЕУРАВНОВЕШЕННОСТЬЮ НА СВЕРХКРИТИЧЕСКИХ СКОРОСТЯХ

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Abstract: *The present paper applies new internal theory to describe dynamics of hinged rotor with quasi-static absence of balance over the whole range of possible velocities. Forces and moments, which effect on rotor and determine its dynamics at different rotational velocities, are under consideration. Notions "virtual support" and "virtual axis of rotor rotation" are introduced. New basic set of equations of rotor dynamics is derived and solved. Rotor dynamics demonstrated to be determined by different sets of equations in different rotation conditions. Analysis of new system of equations is done for cylindrical rotor and disk rotor. Critical rotor velocities are also determined. Dependences are obtained to determine forces, moments and other rotation parameters in subcritical, transient and overcritical conditions of rotation. Equations are derived to determine a shape of shaft bend over the whole range of rotation velocities.*
KEYWORDS*: ROTOR, DYNAMICS, UNBALANCE, SHAFT, SUPPORT*

1. Introduction

Dynamics of outboard rotor with static unbalance has received sufficient attention in [1]. The investigation brought out a property of static unbalance to produce bending moments, which come out of moment rotor unbalance. At the same time the investigation showed that static unbalance could be removed installing one correcting load. Due to the above peculiarities such static unbalance of outboard rotor was called quasi-static unbalance.

By means of the vibration theory, differential equations are derived, which do not take into account the effects of moments of inertia and a shaft bending. So, in particular cases, they result in solution similar to solution of rotor Jeffcott dynamic equations. In common case the equations have no solutions [2].

The inertial theory of rotor dynamics marked the existence of induced moment unbalance of rotor with the availability of static unbalance of outboard rotor and defined a method of giving induced moment unbalance. It enables to derive equations describing rotation of rotor with a hard shaft fixed in hinge and elastic support [3]. However the investigation assumed the absence of a shaft bending.

The inertial theory also enabled to derive dynamic equations of outboard rotor with a flexible shaft with velocities, at which static self-centering of rotor occurs [4]. The equations describe rotor dynamics with quasi-static unbalance subject to the actual effect of induced moment unbalance on rotor position relative to rotation axis.

However rotor dynamics at velocities higher that velocity of static self-centering was not investigated. Rotor dynamic equations which describe moment rotor self-centering and alignment of major inertia axis with rotation axis, are absent.

Moreover, equations to determine a shape of defected shaft axis, which are useful for identification of rotor unbalances, are not derived.

In order to solve problems posed let us consider some peculiarities of rotor dynamics, which are omitted in the known investigation [5].

2. Imaginary Moment and Static Rotor Unbalance

Notion "induced" moment unbalance was not found earlier in scientific and technical literature. So there is need in defining the notion.

Assume that there is rotor with mass m. This rotor has equatorial moment of inertia I_a and axial moment of inertia I_b. The distance from rotor geometric center B to support D equals l. Stiffness coefficient of outboard rotor equals k. Assume that rotor has static unbalance, which is given by shift e of center of mass C from geometric axis Z^I (Fig. 1).

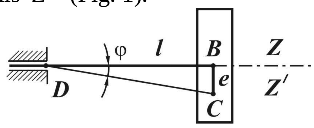

Fig.1 –

The separate rotor has no moment unbalance, as rotor major axis of inertia is parallel to rotor shaft axis. However the tendency of rotating outboard rotor to static self-centering results in DC line alignment with rotation axis Z and so in deflection of geometric rotor axis from rotation axis. At the same time rotor major axis of inertia deflects from rotation axis. It results in moment unbalance characterized by angle φ. Let us call moment unbalance of rotor induced. For the first time the peculiarity was taken into account in the paper [3]. Induced moment unbalance occurs in any rotor, when forced alignment of center of mass results in deflection of rotor major inertia axis from rotation axis.

Similarly, the actual moment unbalance of rotor could result in induced static unbalance of rotor. Such moment unbalance may also be called quasi-moment unbalance.

3. Dynamics of Rotor with Quasi-Static Unbalance at Static Self-Centering Velocities

Dynamics of rotor with quasi-static unbalance at static self-centering velocities is described by set of equations [4]

(1) $$\left[m\omega^2+\frac{5(I_a-I_b)\omega^2}{2l^2}\right](a+e\cos\alpha)-\frac{(I_a-I_b)\omega^2 a}{l^2}-ka=0 ,$$

(2) $$\sin\alpha=a/e ,$$

where a – shift of geometric center of rotor; $k=3EJ/l^3$ – coefficient of shaft stiffness; e – shift of rotor center of mass from geometric axis; α – angle of rotor turning about geometric axis depend upon rotation velocity; EJ – bending stiffness of a straight shaft with uniform section.

Equations derived in operation [4] enable to determine shift of geometric center of rotor from rotation axis a.

At this shift a it is possible to determine lateral force, under which action shaft is bent

(3) $$F_n=ka=3EJa/l^3 ,$$

and angle of rotor inclination to rotation axis φ^* [6]

(4) $$\varphi^* = 3a/2l \, .$$

Limiting minimum value φ^* for rotor under consideration can be determined through angle φ, which defines induced moment unbalance of rotor when $\alpha = 180^\circ$

(5) $$\varphi^* = 3a/2l = 3e/2l = 3\varphi/2 \, .$$

4. Initial Scheme of Outboard Rotor

Assume that rotor is rotated with velocity ω, which is just more than velocity of static rotor self-centering, i.e. the first critical velocity. Assume that at this velocity center of mass is almost aligned with rotation axis and induced moment unbalance only starts affecting rotor dynamics sufficiently. In this case shift of geometric center of rotor from rotation axis Z equals a (Fig. 2). With that $a > e$.

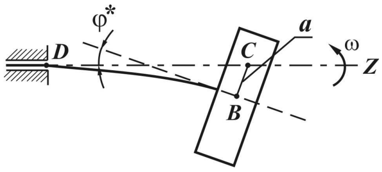

Fig.2 –

By the time angle α of rotor turn about geometric axis equals 180° and angle of rotor inclination to rotation axis equals φ^*. It will be recalled that excess torque M_{kp} is applied to rotor shaft to support rotor position relative to rotation axis. It will be also recalled that complete torque [7] is applied to rotor shaft to support rotor rotation velocity. However the paper does not give equations of these moments as rotor position is investigated relative to rotation axis at specified rotation velocities.

5. Peculiarities of Rotor Rotation

For common case center of mass of rotating rotor takes up definite position relative to rotation axis, which can be changed only under the action of external force. Even chance increase of value a results in unbalance of elastic and centrifugal forces and so in force, which seeks to return rotor to initial position relative to rotation axis. The issues concerning stabilization of center of mass position on rotation axis have been detailed in [8]. Restoring force F_u^B can be determined as follows

(6) $$F_u^B = m\omega^2 y_* \, ,$$

where y_* – forced shift of rotor center of mass relative to rotation axis under the action of external force.

Restoring force can produce, properly, restoring moment M_u^B, which can be written down for outboard rotor as follows

(7) $$M_u^B = F_u^B l \, .$$

However any shift of rotor center of mass is associated with shaft bending. Force F_J^B produced by elastic shift in the rotor attachment point can be determined by the following equation

(8) $$F_J^B = k y_* \, .$$

Force F_J^B can produce, properly, moment M_J^B, which can be written down for outboard rotor as follows

(9) $$M_J^B = F_J^B l \, .$$

Because k depends only on bending stiffness and shift length, comparison of equations (6) (8) (7) (9) shows that restoring force or moment can exceed many times over force or moment produced by elastic shift at high rotor velocities.

So it can be stated that static self-centering of outboard rotor results in the second "virtual" hinged support. Outboard rotor turns out rotor with two supports, one of which has a mobile virtual hinge (Fig. 3).

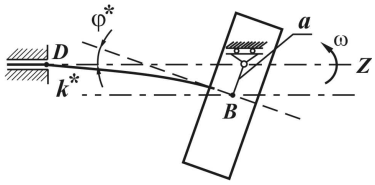

Fig.3 –

Virtual geometric axis, which connects point D to center of virtual hinged support, appears in addition to virtual support.

With that qualitative changes occur. Forces and moments, acting on rotor and a bending shaft relative to support D, start also acting about rotor center of mass.

Shaft section in virtual support position does not move away from rotation axis but changes angular position under the action of moments.

Coefficient of shaft stiffness k fails to play a crucial role in rotor position relative to rotation axis as coefficient of stiffness does not characterize change of rotor shaft inclination angle to rotation axis in virtual hinge.

Actually dynamics of rotor with quasi-static unbalance changes over category of dynamics of rotor with moment unbalance.

However original induced moment unbalance, caused by shift of rotor center of mass and defined by angle φ, does not define initial induced moment unbalance of rotor. In this case original induced moment unbalance is associated with rotor position relative to rotation axis and defined by rotor shaft bending under the action of static unbalance. As a result, original induced moment unbalance of rotor is defined by angle φ^*, which value is the same as for angle φ. Angle φ^* can be defined as an angle between geometric rotor axis and line parallel to rotation axis, which is drawn through geometric center of rotor.

Qualitative changes place new requirements on equations of rotor dynamics. It makes sense to address the peculiarities of dynamics of rotor with moment unbalance.

6. Jeffcott Dynamics of Rotor with Moment Unbalance

For the first time Jeffcott dynamics of rotor with moment unbalance has been examined in [9] and described by the following equations

(10) $$(I_a - I_b)\omega^2(\lambda + \varphi\cos\beta) - k^a\lambda = 0 \, ,$$

(11) $$\sin\beta = \sin\lambda/\sin\varphi = \lambda/\varphi \, .$$

where λ – angle of inclination of geometric rotor axis to rotation axis; φ – angle of inclination of major central inertia axis to geometric rotor axis; β – angle of rotor turn about geometric axis; k^a – coefficient of angular shaft stiffness.

It is evident that set of equations of dynamics of rotor with moment unbalance can be used to describe dynamics of outboard rotor with quasi-static unbalance at velocities higher that static self-centering velocities.

The reason is that in both cases rotor rotation is accompanied by correction of inctination angle of major central inertia axis to rotation axis. In one case inclination angle of major central inertia axis changes due to rotor turn about geometric center. In the other case inclination angle of major central inertia axis changes due to rotor turn about virtual hinge.

Difference in sets of equations is defined only by choice of coefficient of rotor shaft angular stiffness in virtual hinge.

7. Coefficient of Rotor Shaft Angular Stiffness in Virtual Hinge

Let us introduce notion of new coefficient of rotor shaft angular stiffness k^*. Coefficient of angular stiffness connects value of restoring shaft moment with angle of shaft section turn in virtual hinge.

To sufficient accuracy coefficient of angular shaft stiffness of rotor with two supports, one of which is hinge, can be determined by the theory of strength of materials. Depending on value of acting moment, angle of shaft section turn in virtual hinge is determined as follows

(12) $$\lambda = -\frac{Ml}{6EJ}\left(1-3\frac{x^2}{l^2}\right) = \frac{Ml}{3EJ} .$$

Change of shaft section turn angle results in restoring moment acting on rotor

(13) $$M^D = 3\lambda EJ/l = \lambda k^* = k^* \sin\lambda ,$$

where k^* – coefficient of angular stiffness, which is determined as follows

(14) $$k^* = 3EJ/l .$$

8. Dynamic Equation of Outboard Rotor at Velocity Higher Than Critical Velocity of Static Self-Centering

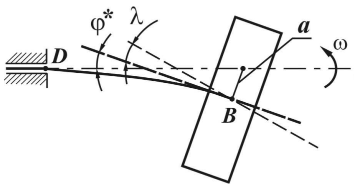

Fig.4 –

Dynamic equation of outboard rotor with quasi-static unbalance (Fig. 4) at velocities higher than critical velocity of static self-centering can be written down as follows

(15) $$\sin\beta = \sin\lambda / \sin\varphi^* = \lambda/\varphi^* ,$$

(16) $$(I_a - I_b)\omega^2(\lambda + \varphi^* \cos\beta) - k^*\lambda = 0 ,$$

where λ – deflection angle of geometric rotor axis from line parallel to rotation axis passing through geometric center of rotor.

Equations, which include only angular values, are the most simplified equations and their application is admissible in accordance with the theory of small displacements.

9. Dynamics of Cylindrical Rotor

For cylindrical rotor we have $I_a - I_b > 0$, $\cos\beta = 1$. In this case from equation (16) we have as follows

(17) $$(I_a - I_b)\omega^2(\lambda + \varphi^*) - \lambda k^* = 0 .$$

From the equation we determine angle λ of rotor shaft section bending

(18) $$\lambda = \sin\lambda = \frac{(I_a - I_b)\omega^2 \sin\varphi^*}{k^* - (I_a - I_b)\omega^2} = \frac{(I_a - I_b)\omega^2\varphi^*}{k^* - (I_a - I_b)\omega^2} .$$

In transient rotation conditions rotor turn about geometric axis by angle β is taken into account. Equation (16) takes the form as follows

(19) $$\left(I_a - I_b)\omega^2(\lambda + \sqrt{\varphi^{*2} - \lambda^2}\right) - \lambda k^* = 0 .$$

As rotation velocity increases rotor turns about geometric axis by some angle β relative to the initial position. When angle equals 90° rotor reaches critical velocity. In this case the equation's member $\sqrt{\varphi^{*2} - \lambda^2}$ is missing and critical velocity is determined by equation

(20) $$\omega_{kp} = \sqrt{k^*/(I_a - I_b)} .$$

Equation (16) can be represented as follows

(21) $$(I_a - I_b)\omega^2(1 + ctg\beta) - k^* = 0$$

Equation (21) shows that rotor turn by angle β does not depend on value of induced moment unbalance.

In supercritical potation condition $\cos\beta = -1$. Equation (16) takes the form

(22) $$(I_a - I_b)\omega^2\left(\lambda - \varphi^*\right) - \lambda k^* = 0 .$$

From equation (22) it is easy to determine angle λ

(23) $$\lambda = \sin\lambda = \frac{(I_a - I_b)\omega^2 \sin\varphi^*}{(I_a - I_b)\omega^2 - k^*} = \frac{(I_a - I_b)\omega^2\varphi^*}{(I_a - I_b)\omega^2 - k^*} .$$

If $\omega \to \infty$, we derive the following equation

(24) $$\lambda = \sin\lambda = \sin\varphi^* = \varphi^* .$$

It follows from dependence that moment rotor self-centering occurs at supercritical velocities changing over critical velocity. With that major axis of rotor inertia coincides with line parallel to rotation axis passing through geometric center of rotor.

10. Dynamics of Disk Rotor

For disk rotor we have $I_b - I_a > 0$, $\cos\beta = 1$. So it is convenient to represent equation (16) in the form

(25) $$(I_b - I_a)\omega^2\left(\lambda + \varphi^* \cos\beta\right) + \lambda k^* = 0 .$$

Equation (25) enables to determine angle λ

(26) $$\lambda = -\frac{(I_b - I_a)\omega^2\varphi^* \cos\beta}{k^* + (I_b - I_a)\omega^2} .$$

Equation (26) shows that angle λ exists only in the case when $\cos\beta = -1$. So equation (25) should be written down in the form

(27) $$(I_b - I_a)\omega^2\left(\lambda - \varphi^*\right) + \lambda k^* = 0 .$$

Due to specific combination of moments of inertia I_a and I_b, the equation describes dynamics corresponding to a case of rotor rotation at supercritical velocities.

It follows from the equation that

(28) $$\lambda = \sin\lambda = \frac{(I_b - I_a)\omega^2\varphi^*}{k^* + (I_b - I_a)\omega^2} .$$

If $\omega \to \infty$, we have

(29) $$\lambda = \sin\lambda = \sin\varphi^* = \varphi^* .$$

Dependence (29) shows that moment rotor self-centering occurs at unlimited increase in velocity.

At moment rotor self-centering, inclination angle of major inertia axis to rotation axis is reduced. However, the phenomenon of rotor self-centering causes increase in machine vibration.

11. Shape of Defected Rotor Shaft Axis

Shape of defected shaft axis of rotor with quasi-static unbalance depends on rotation velocity.

In phase of static rotor self-centering, lateral force F_n, which acts on shaft and depends on rotation velocity, can be determined as follows

(30) $$F_n = ka = 3EJa/l^3 .$$

By means of lateral force, equation of defected shaft axis can be derived for chosen rotation velocity

(31) $$y = \frac{F_n l x^2}{6EJ}\left(3 - \frac{x}{l}\right) = a\frac{x^2}{2l^2}\left(3 - \frac{x}{l}\right).$$

where x – distance from support to chosen shaft section; y – rotor shaft deflection at distance x from support D of outboard rotor.

Angle α of rotor turn about geometric axis enables to determine shaft bending plane position relative to quasi-static unbalance plane depending on chosen velocity (at supercritical velocity angle $\alpha = 180^\circ$). Equation of defected shaft axis enables to construct a shape of defected axis of rotor shaft. In phase of moment rotor self-centering, a shape of defected axis also depends, in addition, on shaft bending at rotor turning in virtual hinge. Thus a shape of defected shaft axis is formed of a set of shapes of defected axis under the action of lateral force F_n and bending moment M_n in virtual hinge.In phase of moment rotor self-centering, bending moment M_n, which acts on shaft in virtual hinge and depends on rotation velocity, can be determined as follows

(32) $$M_n = \lambda k^* = k^* \sin\lambda = 3EJ\lambda/l .$$

By means of moment M_n equation of defected shaft axis can be derived for chosen rotation velocity

(33) $$y = \frac{M_n l x}{6EJ}\left(1-\frac{x^2}{l^2}\right) = \lambda\frac{x}{2}\left(1-\frac{x^2}{l^2}\right).$$

Rotor turn angle β enables to determine shaft bending plane position relative to quasi-static unbalance plane depending on chosen velocity (at supercritical velocity angle $\beta = 180^\circ$).

Equations (31) and (33) of defected shaft axis (without considering rotor mass) enable to construct a shape of defected axis of rotor shaft at velocities before and after moment rotor self-centering. The equations can be combined into one equation

(34) $$y = a\frac{x^2}{2l^2}\left(3-\frac{x}{l}\right) - \lambda\frac{x}{2}\left(1-\frac{x^2}{l^2}\right)\cos\beta .$$

Geometric summation of ordinates y of rotor shaft bending under the action of lateral force F_n and bending moment M_n considering angle β enables to obtain a shape of shaft axis bending at chosen velocity depending on coordinate x.

For example, equation of defected shaft axis of disk rotor after moment self-centering, when $\omega \to \infty$, takes the form Fig. 5.

(35) $$y = e\frac{x^2}{2l^2}\left(3-\frac{x}{l}\right) + e\frac{3x}{4l}\left(1-\frac{x^2}{l^2}\right)$$

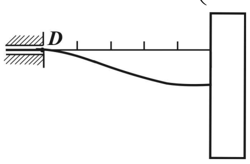

Fig.5 –

12. Support Reaction Forces

Reaction force of support D at rotor velocities to static self-centering is determined by value of lateral force F_n, which can be calculated according to equation (30).

Reaction force of support in phase of moment rotor self-centering is determined by value and direction of lateral force F_n and bending moment M_n. Bending moment M_n can be calculated according to equation (32). It can be determined by the following equation

(36) $$R = \frac{3EJ}{l^3}a - \lambda\frac{3EJ}{l^2}\cos\beta .$$

For example, equation to determine reaction force of outboard disk rotor's support (without considering rotor mass) at significant velocities after moment self-centering takes the form

(37) $$R = 15EJe/2l^3 .$$

13. Results and Discussion

Recent results are obtained as compared to studies conducted earlier [1], [5]. Values and directions of all forces and moments, which act on rotor over the whole velocity range, are determined. Such notions as "virtual support" and "virtual axis of rotor rotation" are introduced.

New dynamic equations of outboard cylindrical and disk rotors at supercritical velocities considering shaft bending are derived as well as equations, which describe rotor position relative to rotation axis in different rotation conditions. Equations are derived, which enable to build a shape of shaft bending at any rotor velocity. It is demonstrated that rotor dynamics is determined according to different sets of equations in different rotation conditions.

Critical velocities are determined. The problems are examined concerning self-centering of cylindrical and disk rotors at supercritical velocities, at which rotor dynamics is defined by quasi-static unbalance.

Excess torque is determined, which requires to keep the associated rotor position about rotation axis.

The investigation show that rotor moments of inertia have the significant effect on rotor dynamics, particularly, on value of critical accelerations. The investigation was conducted without considering dissipative forces that enabled to show real physics of the rotation process associated with the effect of quasi-static unbalance.

The distinctive feature is occurrence of outboard rotor's virtual support upon reaching velocity of static self-centering as well as application of dependence of section deflection angle from shaft bending deflection based on the theory of strength of materials. It enabled to define rotor dynamics in the initial phase that is phase of rotor static self-centering and define dynamics of the final phase of rotor rotation under the action of moment unbalance at supercritical velocities. The effect of moment unbalance at high supercritical velocities results in double shaft bending.

14. Conclusion

The investigation results could be used for designing outboard rotors. They can be applied to determine loads acting in the bearing attachment point of rotor shaft and to determine drive power loss when rotating rotor with quasi-static unbalance.

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