DETERMINATION OF THE OPTIMAL DELIVERY SUPPLY

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Abstract: *This article deals with method of determination of optimal delivery supply of supplier to automotive industry. Today manager's practice lays emphasis on decrease of stocks. The main issues are considered in this topic: the mathematical stochastic stock model to determine the optimal delivery supply.*
KEYWORDS*: STOCK, WAREHOUSE, ABC ANALYSIS, SUPPLY, COST*

1. Introduction

Nowadays automotive industry develops in the Czech Republic. The modern logistic methods (JIT, KANBAN, etc.) are used in the branch of automotive industry. These methods lay demands on suppliers. Main requirements are quality requirement of supplied components, in time supplies and other. This article deals with method of determination of optimal delivery supply of supplier to automotive industry.

2. Importance of stocks for company

Today manager's practice lays emphasis on decrease of stocks. Negative influence of stocks is that tie the capital, spend work and means (stocks have to store, consequently energy costs, service costs, costs of repairing, labour costs, etc.), risk of stock depreciation, risk of unserviceableness or risk of unsaleability (possible reason: change of production program or customer preference). Locked-up capital in stocks is short of the technical development financing, it endangers ability to pay of company and decreases trustworthiness during meetings with business partners and banks (high-stock level warns of bad business management). The stock level should be decreased on the minimum but on the second side it has to ensure sufficient readiness of supplies to customers. It is evident, that both points of view (stock minimization versus high readiness of supplies) are opposed and the company has to choice the compromise.

3. Classification of stocks

Stocks can classify according to many points of view (for example: degree of processing, suitability, function, etc.). Function classification of stocks is the most important from the point of view of operative management.

Function classification of stocks:

turnover stock
safety stock
seasonal pre-stock
strategic stock
technological stock
speculative stock

Turnover stock is the part of stocks which it ensures requirement period between the two supplies. Its level fluctuates during the supply cycle. For that reason it works with the average turnover stock. Its level is half of the supply in ideal case.

Safety stock has the aim to balance stochastic fluctuations partly on the input side to the company (level of supplies and supply interval), partly on the output side from the company (level and interval of draw on stocks).

Seasonal pre-stock balances expected fluctuations in input or output. The company was unable in this period from reason of limited production capacity to satisfy for example strong seasonal consumption of product (for example Christmas). In this case the company begins to make planned stock of product in advance. The company assumes that this planned stock will sell.

Strategic stock ensures function of company during unpredictable incidents (calamity, strike, conflict, etc.).

Technological stock makes in the event of conclusion of production by producer, but the product isn't able to fill consumer demand because it needs to store before application. Some foods (cheese, beer and vine) have to mellow more time.

Speculative stock makes for purpose of achievement of the extra profit by help of suitable purchase (temporary of reduce a price before expected put up a price, purchase for purpose of advantageous future sale).

4. Differential inventory management

In practise it isn't possible, not useful; to pay attention to all stock items the same attention. For that reason it is necessary differential inventory management. Paret's rule helps to determine the most important stock items. Paret's rule says that 80 % consequences follow from 20% possible causes. It means in the issue of stock that 20 % stock items can represent 80 % value of consumption or sale or big part of purchase originates from relatively small number of suppliers. Paret's rule means that inventory management should focus on certain number of the most important objects (for example: stock items, suppliers), which they have controlling influence on total result.

On the basis Paret's analysis we can classify stock assortment to groups according to the criterions. In practise, stock items classify to three groups. This analysis is called ABC analysis.

The inventory management has to arrange stock items downwardly according to value of monitored statistical index (value of consumption or value of sale) in monitored time. Analysed period should be from 12 to 24 months. Shorter period can be misrepresenting by the seasonal influence, in longer period it gets to change of production program and data lose information capability. Next step is the finding of stock items which represent 80 % and 90 % value of consumption or value of sale.

Category A consists of stock items which make 80 % value of consumption or value of sale. They are the most important stock items which monitor daily. The optimal supply and safety stock is determined individually and as exactly as possible.

Category B represent stock items with 15 % value of consumption or value of sale. It means that both categories A and B represent together 95 % value of consumption or value of sale. Stock items of category B are monitored as compared with category A less often and simpler methods are used to manage of them. Value of supply and safety stock is usually higher than stock items of category A.

Category C represent stock items with approximately 5 % value of consumption or value of sale. Very simple methods are used to manage of them. These methods use estimate of average consumption in preceding period.

5. *Stochastic stock model*

We will suppose that intensity of warehouse collection - mean value of withdrawal pieces per unit time is λ. Stocking products arrive to the warehouse after m pieces which are the most number of products in the warehouse. If K is total warehouse capacity then m=K. Intensity of supply is μ. Impulse for transposition supply into the warehouse is on condition empty warehouse, consequently it replenishes always the empty stock.

The mean time between warehouse collections is the inverse value of intensity of warehouse collection $1/\lambda$, similarly mean value between two supplies is the inverse value of intensity of supplies $1/\mu$.

By the help of queuing theory derives transition probability. Transition probabilities means that the system goes over from system state i to system state (i+1) or (i-1) or stays in system state i (P. M. Morse in work Queues, Inventories and Maitenance deals with model).

Probabilities p_i of system state i (the system has just i system states) were derived on condition stationary distribution.

$$p_1 = p_2 = ...= p_K = \frac{\mu}{\lambda} p_0$$

$$p_0 = \frac{1}{1 + K\dfrac{\mu}{\lambda}}$$

We will optimise quantity m=K, on condition profit is criterion of optimization. We will suppose that profit per sold unit is g, hold costs per time T on unit are c1 and costs of transposition supply m=K units are c0. If K is enough high, it is possible that the continuous function approximates development of profit function.

Profit function is:

$$Z(K) = g \cdot L_s - c_1 - c_0 p_0$$

where

$$L_s = T(1 - p_0) = \frac{\lambda \cdot T \cdot K}{K + \lambda \cdot T}$$

L_s is average number of warehouse collection per time T.

We can calculate the average stock:

$$I = \sum_{n=0}^{K} n \cdot p_n = \frac{K^2 + K}{2(K + T\lambda)}$$

After appointment:

$$Z(K) = \frac{g \cdot T \cdot \lambda \cdot K - \frac{c_1}{2} \cdot (K^2 + K) - c_0 \cdot T \cdot \lambda}{K + T \cdot \lambda}$$

First derivative of profit function is zero for the determination of maximum profit and then we can deduce level of supply:

$$K_{1,2} = -T \cdot \lambda \pm \sqrt{(T \cdot \lambda)^2 - T \cdot \lambda + \frac{2c_0}{c_1} \cdot T \cdot \lambda + \frac{2}{c_1} \cdot g \cdot (T \cdot \lambda)^2}$$

Because the positive solution is convenient to the problem and other quantities are negligible in relation to c0/c1 a g/c1.It is possible to simplify the relation. We will suppose that minimum size of warehouse equals the optimal supply. We can determine level of supply m0 and warehouse capacity as follows:

$$m_0 = K_0 \approx \sqrt{2T\lambda \cdot \frac{g \cdot T \cdot \lambda + c_0}{c_1}}$$

In case that criterion of optimization is minimization of hold costs so cost function is following:

$$N(K) = \frac{\frac{1}{2} c_1 \left(K^2 + K\right) + c_0 T\lambda}{K + T\lambda}$$

Optimal level of supply (N´(K)=0) will be in the same case as optimal criterion of profit:

$$m_0' = K_0' \approx \sqrt{\frac{2T\lambda c_0}{c_1}}$$

Till now we supposed the condition empty warehouse, consequently it replenishes always the empty stock. In practise it isn’t always possible. We have to define number of units in the warehouse, where stock level mustn't drop it like safety stock D. Impulse for the supply will be achievement of safety stock. Then we order m= K-D units to replenish stock in the warehouse on level K.

We deduce forms of probabilities for the stationary distribution of system probabilities (by help of queuing theory).

$$2\mu p_0 = \lambda p_1 \qquad 0 < n \le D$$

$$(\mu+\lambda)p_n = \lambda p_{n+1}$$

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$$(\mu+\lambda)p_D = \mu p_0 + \lambda p_D \qquad D < n \le m$$

$$\lambda p_n = \lambda p_{n+1}$$

$$\lambda p_n = \lambda p_{n+1} + \mu p_{n-m} \quad m < n \le K$$

$$p_K = \mu p_K - m$$

$$\sum_{n=0}^{K} p_n = 1$$

If is $\mu = 1/T$, we can determine probabilities of system state:

$$p_n = \frac{2}{T\lambda}\left(1+\frac{1}{T\lambda}\right)^{n-1} p_0 \qquad 0 < n \le D$$

$$p_n = \frac{2}{T\lambda}\left[\left(1+\frac{1}{T\lambda}\right)^{D} - \frac{1}{2}\right] p_0 \qquad D < n \le m$$

$$p_n = \frac{2}{T\lambda}\left[\left(1+\frac{1}{T\lambda}\right)^{D} - \left(1+\frac{1}{T\lambda}\right)^{n-m-1}\right] p_0 \qquad m < n \le K$$

$$p_0 = \frac{(T\lambda)^{D+1}}{2m(1+T\lambda)^{D} + (T\lambda + D - m)(T\lambda)^{D}}$$

On condition profit function we can determine optimal level of supply m0, if first derivative of profit function is zero according to m. If the warehouse capacity is K then K0=K-D is the optimal warehouse capacity over safety stock.

6. Simulation

This model of determination of optimal delivery supply was used in the company. Numbers are fictitious and illustrative for reason confidentiality.

condition m = K
m = K= 5000 pcs.
T =1 day
λ = 100 pcs./day
μ = 150 pcs./day
g = 1200 Kč
c1 = 35 Kč/day
c0 = 15 Kč

1) We will optimise quantity m=K, on condition profit is the criterion of optimization.

$$m_0 = K_0 \approx \sqrt{2T\lambda \cdot \frac{g \cdot T \cdot \lambda + c_0}{c_1}} = \sqrt{2 \cdot 1 \cdot 100 \cdot \frac{1200 \cdot 1 \cdot 100 + 15}{35}} = \sqrt{685714{,}71} = 828\, pcs.$$

2) We will optimise quantity m=K, on condition minimization of hold costs is the criterion of optimization.

$$m_0' = K_0' \approx \sqrt{\frac{2T\lambda c_0}{c_1}} = \sqrt{\frac{2 \cdot 1 \cdot 100 \cdot 15}{35}} = \sqrt{\frac{3000}{35}} = \sqrt{85{,}71} = 9{,}26 \approx 10\, pcs.$$

7. Conclusion

The stock level should be decreased on the minimum but on the second side it has to ensure sufficient readiness of supplies to customers. It is evident, that both points of view (stock minimization versus high readiness of supplies) are opposed and the company has to choice the compromise. It was proposed the mathematical stochastic stock model to determine the optimal delivery supply.

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