MATHEMATICAL MODELING OF PARTICLES HEATING IN TRANSFERRED DC "TUBULAR" PLASMA ARC

МАТЕМАТИЧЕН МОДЕЛ НА НАГРЯВАНЕТО НА ЧАСТИЦИ В ПРЕХВЪРЛЕНА ПЛАЗМЕНА ДЪГА "ТРЪБА"

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Abstract: *A mathematical model of disperse phase heating in a "tubular" plasma arc volume is presented. The transferred DC "tubular" plasma arc is considered as a heterogeneous flow with uniform particle distribution across its volume. Temperature and concentration fields in the arc for different particles' mass flow rates for two different gas velocities are determined. The numerical simulation by means of the newly developed mathematical model permits to obtain the temperature, velocity, electromagnetic and concentration fields. The change of average particle temperature as an arc length function is presented.*

Keywords: *MATHEMATICAL MODELING, TRANSFERRED DC "TUBULAR" PLASMA ARC, PARTICLE HEATING*

1. *Introduction*

Developing a technology and equipment using plasma arc involves high expenses and finally depends on optimal choice of specific technological parameters that cannot be derived from other existing technologies and equipment.

This necessitates development of mathematical models describing the plasma heating processes and their application to technological parameter determination and consequent optimization.

The plasma arc is presented as a magneto-hydrodynamic turbulent fluid flow using the magnetic gas dynamics equations. These models describe very well the electric arc and attendant physical processes by a system of differential equations including the equation of momentum, the heat transfer and the electromagnetic field influence as well as the transport characteristics. Numerical simulation by means of mathematical models permits to obtain the temperature, velocity, electromagnetic and concentration fields of the plasma arc[1].

Turbulent flows and turbulent heat transfer can be modeled at different levels of complexity and accuracy. In this case use of the K-ε turbulence model is the most appropriate [2]. Such a model gives possibilities of describing laminar and turbulent flows and removes limitations of gradient models.

2. *DEVELOPMENT OF A MATHEMATICAL MODEL OF PARTICLE HEATING IN TRANSFERRED DC "TUBULAR" PLASMA ARC.*

Heat transfer and disperse material movement in high temperature flow are a complex set of interrelated phenomena. They depend on material properties and ambiance as well as parallel processes such as vaporization, phase transformation, radiation, dissociation and recombination of components etc.

For many plasma metallurgical technologies it is very important to obtain the rate of particles heating in the plasma arc volume between the cathode and anode. Through variation of the electric, geometric and gas-dynamic parameters of stable burning well-configured "tubular" DC plasma arc, the optimal values that ensure technological heating of particles in the different plasma metallurgical processes can be obtained.

2.1. Mathematical model of transferred DC "tubular" plasma arc.

The mathematical models describing the plasma arc particles heating are usually based on equations including conservation equation of mass and energy transport equation [3]. Since the plasma arc is a conductive fluid, our model included another equation describing electromagnetic field and specific mass flow rate. The aim is to present the differential equations in a form suitable for numerical solution. To solve a mathematical problem, initial and boundary conditions and thermodynamic and transport properties are required.

The differential transport equations can be expressed in a generalized form as:

$$\operatorname{div}\left(\rho \Phi \vec{v}\right) = \operatorname{div}\left(\Gamma_\Phi \operatorname{grad}(\Phi)\right) + S_\Phi , \qquad (1)$$

where the coefficients are given in Table 1

(Table 1) Coefficients of the differential transport equation

Φ	Γ_Φ	S_Φ
1	0	$m_p g$
v_z	$\mu_{eff} = \mu_l + \mu_t$	$J_r B - \frac{\partial p}{\partial z} + m_p g$
v_r	μ_{eff}	$-J_z B - \frac{\partial p}{\partial r}$
K	$\mu_l + \frac{\mu_t}{\sigma_K}$	$2\mu_t \left[\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{v_r}{r}^2 + \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)^2 \right] - \rho\varepsilon$
ε	$\mu_l + \frac{\mu_t}{\sigma_\varepsilon}$	$2C_{\varepsilon 1}\mu_t \frac{\varepsilon}{K}\left[\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{v_r}{r}^2 + \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)^2 \right] - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{K}$
T	$\frac{\lambda_{eff}}{c_p}$	$\frac{1}{c_p}\left[\left(\frac{J_z^2 + J_r^2}{\sigma}\right) - S_R - \frac{\partial (c_p T) m_p}{\partial \tau}\right]$
B	$\frac{\rho}{\mu_0 \sigma}$	0
f	$\frac{\mu_t}{0,9}$	0

where:

$$Z = J_r B + m_p g; \qquad J_r = -\frac{1}{\mu_0}\frac{\partial B}{\partial z}, \qquad (2)$$

$$R = -J_z B, \qquad J_z = \frac{1}{\mu_0 r}\frac{\partial}{\partial r}(rB) \cdot \tag{3}$$

$$\mu_t = \rho C_D \frac{K^2}{\varepsilon} \tag{4}$$

$$\lambda_{eff} = \lambda_l + \lambda_t, \qquad \lambda_t = (1{,}4 \div 2)\mu_t \tag{5}$$

The heat losses S_R include arc radiation losses S_{RA} and particles radiation losses S_{Rp}:

$$S_R = S_{RA} + NS_{Rp} = S_{RA} + N\varepsilon_p \sigma_0 T_p^4 F_p, \tag{6}$$

The particles heating results from radiation and convective heat transfer. The particle energy balance equation can be presented as:

$$m_p c_p \frac{\partial T_p}{\partial \tau} = \alpha_\Sigma F_p (T_A - T_p) \tag{7}$$

The summary heat transfer coefficient α_Σ in equation (7) includes the convective and radiation particle heating ($\alpha_\Sigma = \alpha + \alpha_R$). The convective heat transfers coefficient - α can be obtained by the Ranz-Marshall equation [5]:

$$Nu = \frac{\alpha d_p}{\lambda} = 2 + 0{,}6 Re^{0{,}5} Pr^{0{,}33}, \tag{8}$$

The radiation heat transfer coefficient α_R is:

$$\alpha_R = \frac{q}{T_A - T_P}, \tag{9}$$

2.2. Initial and boundary conditions.

The system of differential equations, describing the transferred DC "tubular" plasma arc includes equation (1). The initial and boundary conditions determine the specific solution of the system. The initial conditions give the parameter values or redistribution at the moment τ=0. The boundary conditions defined the calculation domain and give all boundary parameters.

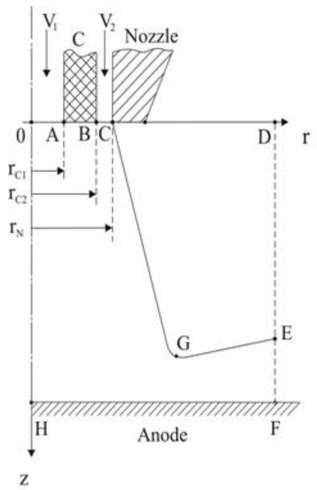

Figure 1.Computation domain and boundary conditions.

3. NUMERICAL SOLUTION

The numerical solution of equation (1) includes the following steps:

- choice of a computing grid;
- approximation of differential equations;
- development of a computing algorithm;
- program realization.

The differential equation system includes the conservation equation mass continuity, momentum and energy; the magnetic induction equation and the specific concentration of solid phase. The numerical simulation uses the control volume method. The computational scheme chosen for solving the problem includes finite differential approximation of the differential equations and boundary conditions. As a results a system of equations is obtained. An iteration procedure for solving has been carried out.

3.1. Numerical simulation and results

For the effective heating of the disperse phase in "tubular" plasma arc volume, the equal velocity of both gasses is most appropriate for uniform particles heating. From a technical and economical point of view the minimal plasma gas flow rate is most advisable.

A series of calculations with different gas velocities are made. As a result of the calculations, the temperature distribution, concentration fields and variation of average particle temperature as an arc length function are predicted. Fig. 2 shows the temperatures distribution for the plasma arc axis cross section at three different gas velocities: v_1=v_2=12 m/s, v_1=v_2=20 m/s and v_1=v_2=28 m/s. The calculations are made for the following plasma arc parameters: I=250 A, U=85 V, L_A=100 mm. The values of heat physical characteristics are presented in [6, 7]. The comparison between the specific concentration fields of each particle mass flow rate in the arc is illustrated in Fig. 3. The results are obtained at a constant voltage. Actually, with the increase of the gas velocities, the effective arc cross section area decreases, which brings about an increase of voltage. The initial values are presented in Table 3.

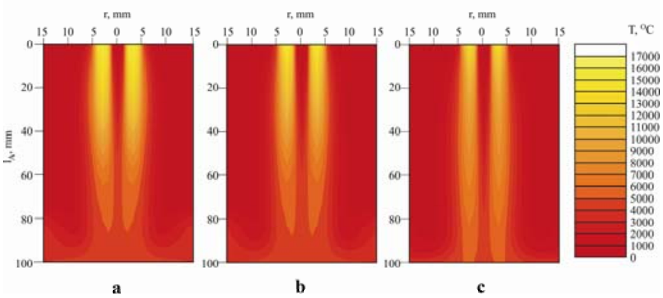

Figure 2. Temperature distribution for the plasma arc axis cross section at gas velocities: **a** - v_1=v_2=12 m/s, **b** - v_1=v_2=20 m/s and **c** - v_1=v_2=28 m/s.

Table 3 Values of gas velocities and particles mass flow rate

v_1=v_2,m/s	Particles mass flow rate ,g/min			
12	5	20	30	50
Voltage, V	90	112	88	70
20	5	20	30	50
Voltage, V	95	115	90	78

From the concentration fields presented in Fig. 3 it can be seen that, as a result of electromagnetic forces, the plasma arc attracts the electrical conductive particles. The isoconcentration lines maximum is shifted from the symmetry axis. With increase of the mass flow rate, displacement of the deviation from the nozzle is observed.

Table 2. Boundary conditions							
Area from **Fig.1.**	v_z	v_r	K	ε	T	B	f
OH (r=0)	-	**0**	$\frac{\partial K}{\partial r}=0$	$\frac{\partial \varepsilon}{\partial r}=0$	$\frac{\partial T}{\partial r}=0$	**0**	$\frac{\partial f}{\partial r}=0$
OA (z=0, 0<r<r$_{C1}$)	$v_z=\frac{V_1}{\pi r_{C1}^2}$	**0**	$K=0{,}02\frac{v_z^2}{\rho}$	$\varepsilon=\frac{C_D K^{1,5}}{0{,}02 r_{C2}}$	$T=T_g$	**0**	**f**
AB (z=0, r$_{C1}$<r<r$_{C2}$)	$v_z=\left(\frac{5\mu_0}{6\rho}\right)^{\frac{1}{2}} r_e J_C$	**0**	$K=0{,}02\frac{v_z^2}{\rho}$	$\varepsilon=\frac{C_D K^{1,5}}{0{,}02 r_{C2}}$	$T=T_A$	$B=\frac{\mu_0 I r}{2\pi r_{C2}^2}$	**0**
BC (z=0, r$_{C2}$<r<r$_N$)	$v_z=\frac{V_2}{\pi\left(r_N^2-r_{C2}^2\right)}$	**0**	$K=0{,}02\frac{v_z^2}{\rho}$	$\varepsilon=\frac{C_D K^{1,5}}{0{,}02 r_{C2}}$	$T=T_g$	$B=\frac{\mu_0 I r}{2\pi r}$	**0**
CD (z=0, r>r$_N$)	**0**	**0**	$\frac{\partial K}{\partial r}=0$	$\frac{\partial \varepsilon}{\partial r}=0$	$T=T_{amb}$	$B=\frac{\mu_0 I r}{2\pi r}$	**0**
DE (r=R)	$\frac{\partial v_z}{\partial r}=0$	$\frac{\partial v_r}{\partial r}=0$	**0**	**0**	$\frac{\partial T}{\partial r}=0$	$B=\frac{\mu_0 I r}{2\pi r}$	**0**
EF (r=R)	$\frac{\partial v_z}{\partial r}=0$	$\frac{\partial v_r}{\partial r}=0$	**0**	**0**	$\frac{\partial T}{\partial r}=0$	$B=\frac{\mu_0 I r}{2\pi r}$	**0**
FH (z=l$_A$)	**0**	**0**	**0**	**0**	$T=T_A$	$\frac{\partial B}{\partial z}=0$	**f**

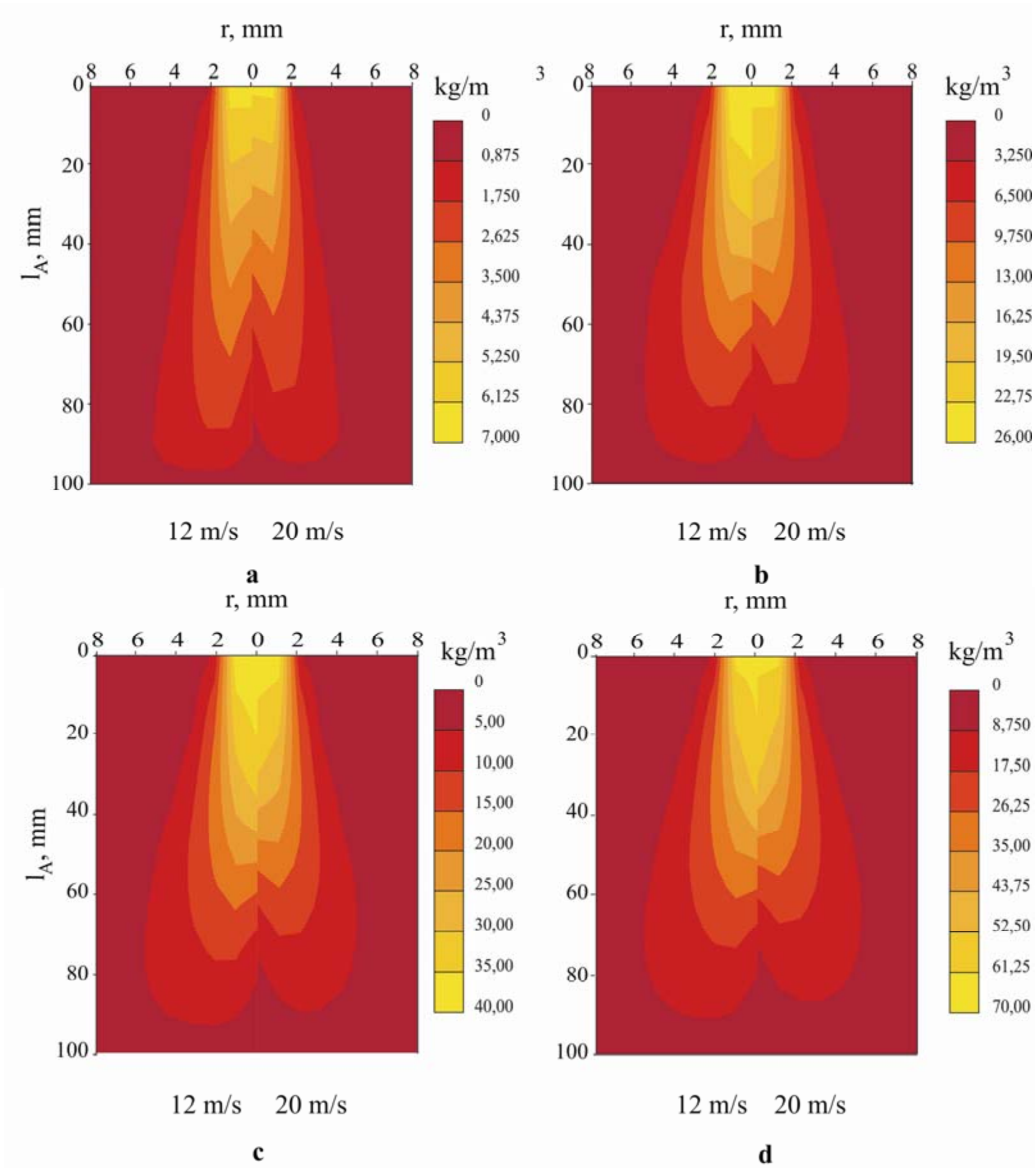

Figure 3. Specific concentration distribution of each particle mass flow rate and two gas velocities: a - 5 g/min, b - 20 g/min, c - 30 g/min and d-50 **g/min.**

These results corroborate the conjecture above about the minimal plasma gas flow rate. The change of average particle temperature as an arc length function is presented in Fig. 4 at a gas flow velocities $v_1=v_2=12$ m/s. As it is shown the particle heating curves at mass flow rate 5, 20 and 30 g/min have a maximum, which is most distinct in the case of 20 g/min. This is a result of irregular temperature distribution along the arc length. With the increase of a mass flow, the effective particle heating decreases. It can be explained by the particle heat shield against plasma arc radiation.

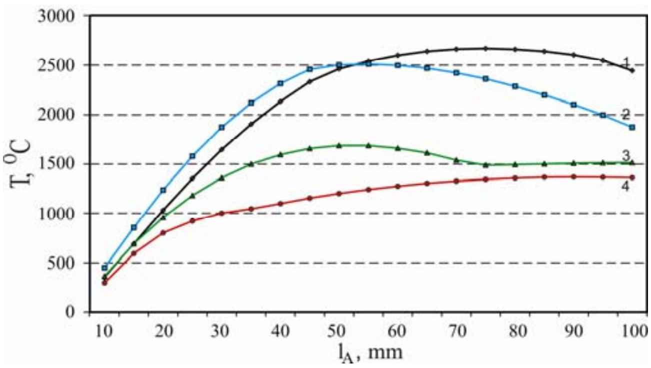

Figure 4. Average particle temperature as an arc length function at a gas flow velocities $v_1=v_2=12$ m/s: 1- 5 g/min, 2- 20 g/min, 3- 30 g/min, and 4- 50 g/min.

As seen from the Fig. 4 the rate of particle temperature increase in the high temperature zone of 20g/min is higher than that at 5 g/min, as a result of the higher arc power and temperature (for 20 g/min) respectively. The numerical simulation results show that at 30 g/min mass flow the final particle temperature reaches near melting point temperature, which is confirmed by the experimental result [7,8].

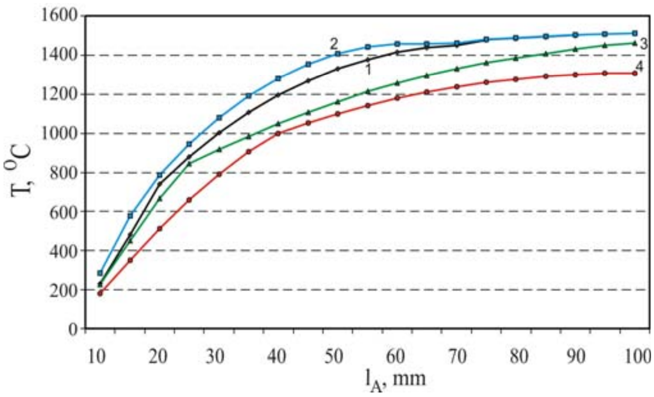

Figure 5. Average particle temperature as an arc length function at a gas flow velocities $v_1=v_2=20$ m/s: 1- 5 g/min, 2- 20 g/min, 3- 30 g/min, and 4- 50 g/min.

At 20 m/s plasma gasses velocity (Fig. 5) the particle temperature increases smoothly (without maximums). As shown at 5 and 20 g/min the particle temperature increases to nearly melting point temperature, which is experimentally registered [8]. In the case of 30 and 50 g/min the final particle temperature is lower than the melting point temperature. Despite the increase of the gas velocities from 12 m/s to 20 m/s, the voltage, respectively the arc power increases because of the pinching of plasma, the particles temperature is lower as a result of their higher velocity and the short time of stay.

4. *CONCLUSIONS*

A mathematical model of disperse phase heating in a "tubular" plasma arc volume is presented. The transferred DC "tubular" plasma arc is considered as a heterogeneous flow with uniform particle distribution across its volume. The temperature and concentration fields in the arc for different particles mass flow rates and two different gas velocities are determined. The numerical simulation by the developed new mathematical model permits to obtain the temperature, velocity, electromagnetic and concentration fields. The results obtained from numerical calculation, and experimental from the disperse material heating investigation, are in good agreement.

List of symbols

σ - electrical conductivity, S/m;
ρ - mass density, kg/m^3;
τ - time, s;
σ_ε - K-ε model constant for ε;
σ_K - K-ε model constant for K;
λ_l – laminar thermal conductivity, W/mK;
μ_l - laminar viscosity, Pa.s;
λ_t - turbulent thermal conductivity, W/mK;
μ_t – turbulent viscosity coefficient, Pa.s;
λ_{eff} - effective thermal conductivity, W/mK;
μ_{eff}– effective viscosity coefficient, Pa.s;
B – electromagnetic induction, T;
$C\varepsilon_1$ – K-ε model constant;
$C\varepsilon_2$ – K-ε model constant;
C_D-dissipation rate constant (C_D=0,.09)[4];
c_p – specific heat, J/kg.K;
f – specific, kg/kg;
F_p – particles surface, m^2.
g – acceleration of gravity, m/s^2;
J_r – radial current density, A/m^2;
J_z – axial current density, A/m^2;
m_p – specific particle mass, kg/m^3;
N – number of particles per volume;
p - pressure, Pa;
Pr is a Prantdl number.
q is a resultant radiation heat flow, report on and the radiation losses from the particles.
r – radial coordinate, m;
R – radial mass force, N/m^3;
Re is a Reynolds number;
T_p –temperature of particles, K;
v_r - radial gas velocity, m/s;
v_z - axial gas velocity, m/s;
z – axial coordinate, m;
Z – axial mass force, N/m^3;
ε_p is a particles emission coefficient;
μ_0 - magnetic permeability of vacuum (4πx10^{-7} H/m);
σ_0 – Stefan-Boltzmann coefficient, (5,67x10^{-8} W/m^{2}K^4);

5. *REFERENCES*

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