DETERMINATION OF NATURAL FREQUENCIES OF A CAR TAKING INTO CONSIDERATION INFLUENCE OF A POWER UNIT

ОПРЕДЕЛЕНИЕ СОБСТВЕННЫХ ЧАСТОТ АВТОМОБИЛЯ С УЧЕТОМ ВЛИЯНИЯ СИЛОВОГО АГРЕГАТА

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Abstract: *In the paper dynamic behaviour of one arm independent suspension for a car is investigated as take account influence of a power unit. For describing the mechanical system is used mathematical model based on vector-matrix algebra. The results from numerical experiments show equation of movement the suspended and unsuspended masses and theirs natural frequencies.*
KEYWORDS*: MATHEMATICAL MODELS, VIBRATIONS, NATURAL FREQUENCIES, RIDE COMFORT*

1. Introduction

The purpose of the suspension of the power unit is to reduce its vibration and transmit dynamic loads to the frame. Of the construction of the suspension depends not only its reliability, but also to a large extent the reliability of the power unit, the frame and the cab. This is due to the interconnected loops through dynamic reactive systems, transmission and construction, which is attached to the internal combustion engine [1].

In selecting the location and the elasticity of the supports necessary to first take into account the values of the natural frequencies of the power unit - they must be within the prescribed (Table 1). The greater is the ratio of frequencies and forced vibrations of its own power plant, the smaller the oscillation of the power unit and the whole car. Crucial natural frequency of individual units and components of the car does not coincide with each other. Indicative range of natural frequencies of the main units and components of the car is shown in Table 1 [2].

Table 1

Suspended mass (body)	1-2 Hz
Seat with the driver (suspended)	1,3-3 Hz
Cabin (suspended)	3-8 Hz
Unsuspended mass	5-13 Hz
Power unit	5-30 Hz
First form of the torsional oscillations of a car body	15-30 Hz
First torsional oscillations form of a power unit	80-200 Hz
First torsional oscillations form of a transmission	60-150 Hz
Tires	50-200 Hz
Panels of the cab and body	> 50 Hz

The aim of this work is to determine the natural frequencies of the suspended and unsuspended masses and those of the power unit of a car with arm suspension, taking into account the influence of the elasticity of tires, the main elastic elements (springs) and elastic elements of the power unit(mountings). For this purpose are used the methods of the vector mechanics, numerical experiments were conducted with MATLAB.

2. Three-dimensional mathematical model

The behavior of the car is described using the three-dimensional mathematical model. The advantage of this scheme is that it is possible to examine the relocation of the car on the axes Ox, Oy and Oz of the coordinate system located in the center of its mass (i.e. all degrees of freedom) which is a prerequisite for high accuracy in computation process [2]. Scheme of the model is shown in Figure 1.

The system under consideration consists suspended mass, unsuspended mass and masses over suspended. The suspended masses include the masses of the elements of the car, passengers and load. In the center of gravity is fixed local coordinate system attached $O_0x_0y_0z_0$. The suspension is implemented as a tire, arm, axle and other components are combined in one element which is hinged to the suspended masses. Each of these elements is fixed to local coordinate system, respectively $O_1x_1y_1z_1$, $O_2x_2y_2z_2$, $O_3x_3y_3z_3$, $O_4x_4y_4z_4$. The masses over suspended include the mass of the power unit (engine and gearbox). In its mass center is fixed local coordinate system Odxdydzd. In the equilibrium position the axis of all coordinate systems are parallel. All displacements of local coordinate systems are given to the absolute coordinate system $O_Ax_Ay_Az_A$.

For systems of Fig. 1 make the following assumptions [3], [4]:

-elements of the system are solids;

-anti-roll bars are massless and their stiffness is regarded as equivalent spring connected to the arms at point to a distance L_{sf} of the joint (hinge) of the front axle and L_{sb} of the joint of the rear axle;

-give an account damping and elastic properties of the main elements c_{rf}, c_{rb}, β_{rf}, β_{rb}, respectively, springs and shock absorbers the front and the rear axle, and the elasticity of the tire c_{gz} and those on wich the mountings of the power unit c_d and β_d;

-elastic and damping elements have linear characteristics;

-system is placed in a equilibrium position as the centers of gravity to the wheels lie on a horizontal axis. O_1y_1 axis coincides with the axis O_2y_2, and O_3y_3 axis coincides with O_4y_4.

For generalized coordinate systems are adopted:

- z_0 - linear displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ on axis O_z;

-elastic and damping elements have linear characteristics;
-system is placed in a equilibrium position as the centers of gravity to the wheels lie on a horizontal axis. O_1y_1 axis coincides with the axis O_2y_2, and O_3y_3 axis coincides with O_4y_4.

For generalized coordinate of the system are accepted:

- z_0 - linear displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ on axis O_z;
- φ_0, ψ_0 - angular displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ respectively around the axes O_x and O_y;
- φ_1 - angular displacement around the axis O_1x_1 of the coordinate system $O_1x_1y_1z_1$;
- φ_2 - angular displacement around the axis O_2x_2 of the coordinate system $O_2x_2y_2z_2$;
- ψ_3 - angular displacement around the axis O_3y_3 of the coordinate system $O_3x_3y_3z_3$;
- ψ_4 - angular displacement around the axis O_4y_4 of the coordinate system $O_4x_4y_4z_4$;

- z_d - linear displacement of the local coordinate system $O_dx_dy_dz_d$ to absolute $O_Ax_Ay_Az_A$ on axis O_z;

- φ_d - angular displacement around the axis O_dx_d of the coordinate system $O_dx_dy_dz_d$;

- ψ_d - angular displacement around the axis O_dy_d of the coordinate system $O_dx_dy_dz_d$.

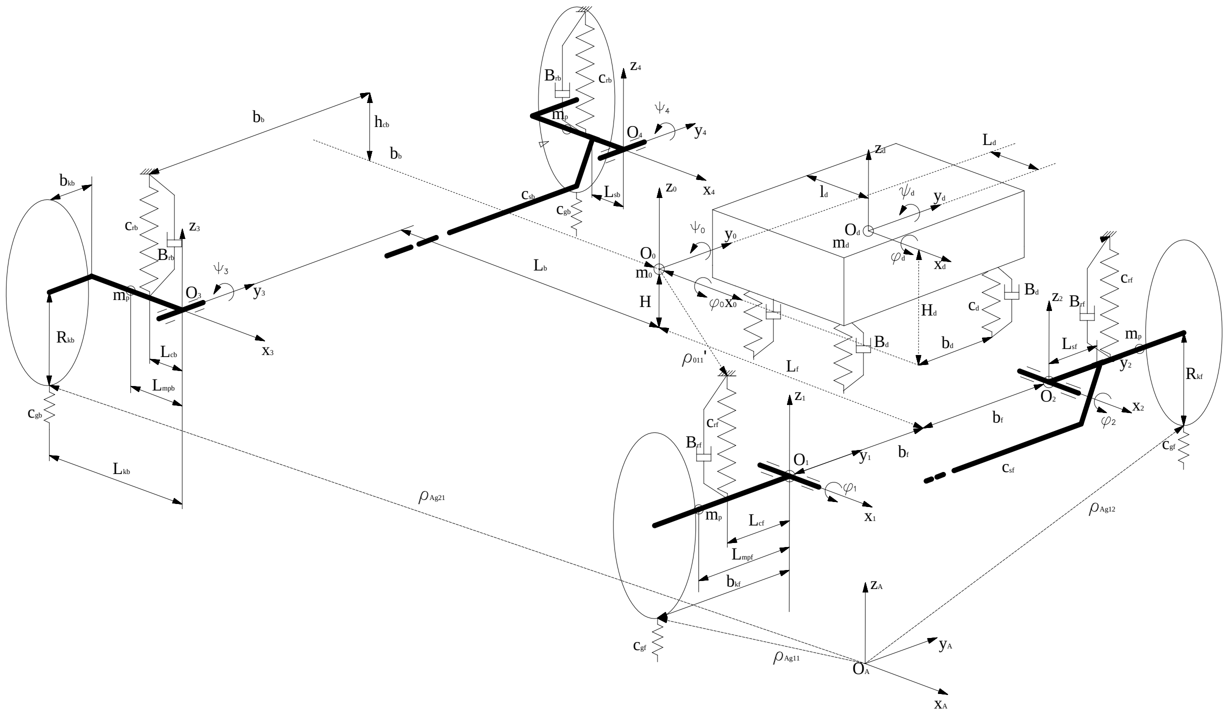

Figure 1. Kinematic scheme of a car with front transverse and rear longitudinal arms with power unit.

To find laws of motion in the absolute coordinate system $O_Ax_Ay_Az_A$ is necessary to define the transition matrices of each local coordinate systems to the absolute [5], [6].

- matrix of transition from $O_0x_0y_0z_0$ to $O_Ax_Ay_Az_A$:

$$T_0^A = \begin{bmatrix} \cos\psi_0 & 0 & -\sin\psi_0 & 0 \\ -\sin\varphi_0.\sin\psi_0 & \cos\varphi_0 & -\sin\varphi_0.\cos\psi_0 & 0 \\ \cos\varphi_0.\sin\psi_0 & \sin\varphi_0 & \cos\varphi_0.\cos\psi_0 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x0 and y0 are zero because is consider only linear oscillation on axis Oz, i.e. only vertically;

- matrices of transition from $O_1x_1y_1z_1$, $O_2x_2y_2z_2$, $O_3x_3y_3z_3$, $O_4x_4y_4z_4$, to $O_0x_0y_0z_0$ have a type:

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos\varphi_1 & -\sin\varphi_1 & -b_f \\ 0 & \sin\varphi_1 & \cos\varphi_1 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos\varphi_2 & -\sin\varphi_2 & b_f \\ 0 & \sin\varphi_2 & \cos\varphi_2 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} \cos\psi_3 & 0 & -\sin\psi_3 & -L_b \\ 0 & 1 & 0 & -b_b \\ \sin\psi_3 & 0 & \cos\psi_3 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} \cos\psi_4 & 0 & -\sin\psi_4 & -L_b \\ 0 & 1 & 0 & b_b \\ \sin\psi_4 & 0 & \cos\psi_4 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

-matrix of transition from $O_dx_dy_dz_d$ to $O_0x_0y_0z_0$ is:

$$T_d^0 = \begin{bmatrix} 1 & 0 & -\psi_d & L_d \\ -\varphi_d.\psi_d & 1 & -\varphi_d & 0 \\ \psi_d & \varphi_d & 1 & H_d - z_d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The components of the angular velocity of sprung masses are set in advance:

$$\omega_{0x}^A = \dot{\varphi}_0$$
$$\omega_{0y}^A = \dot{\psi}_0$$
$$\omega_{0z}^A = 0$$

Otherwise however is the issue of determining the angular velocities of the arms where there is transmission and relative motion.

$$\underline{\omega_i^{0*}} = -T_i^0 \dot{T}_i^{0(T)} = \dot{T}_i^0 T_i^{0(T)}$$

ω_i – the angular velocity of the i-th unit to the absolute coordinate system is equal to:

$$\underline{\omega_i^{0*}} = \begin{bmatrix} 0 & -\omega_{iz} & \omega_{iy} \\ \omega_{iz} & 0 & -\omega_{ix} \\ -\omega_{iy} & \omega_{ix} & 0 \end{bmatrix}$$

After multiplying the matrices and simplify the resulting expressions for the components of the angular velocity of the arms and the three axes are obtained:

- front right arm:

$$\omega_{1x}^A = \dot{\varphi}_0 + \dot{\varphi}_1 \cos\psi_0$$
$$\omega_{1y}^A = \dot{\psi}_0 \cos\varphi_0 + \dot{\varphi}_1 \sin\psi_0 \sin\varphi_0$$
$$\omega_{1z}^A = \dot{\varphi}_1 \sin\psi_0 \cos\varphi_0 - \dot{\psi}_0 \sin\varphi_0$$

After removal of terms of a higher order is received:

$$\omega_{1x}^A = \dot{\varphi}_0 + \dot{\varphi}_1$$
$$\omega_{1y}^A = \dot{\psi}_0$$
$$\omega_{1z}^A = 0$$

Similarly to determine the angular velocities of the other arms:

- front left arm:

$$\omega_{2x}^A = \dot{\varphi}_0 + \dot{\varphi}_2$$
$$\omega_{2y}^A = \dot{\psi}_0$$
$$\omega_{2z}^A = 0$$

- rear right arm:

$$\omega_{3x}^A = \dot{\varphi}_0$$
$$\omega_{3y}^A = \dot{\psi}_0 + \dot{\psi}_3$$
$$\omega_{3z}^A = 0$$

- rear left arm:

$$\omega_{4x}^A = \dot{\varphi}_0$$
$$\omega_{4y}^A = \dot{\psi}_0 + \dot{\psi}_4$$
$$\omega_{4z}^A = 0$$

- for power unit:

$$\omega_{dx}^A = \dot{\varphi}_0 + \dot{\varphi}_d$$
$$\omega_{dy}^A = \dot{\psi}_0 + \dot{\psi}_d$$
$$\omega_{4z}^A = 0$$

The kinetic energy of the system is:

$$\begin{aligned}
\mathrm{T} &= \frac{1}{2} m_0 \dot{z}_0^2 + \frac{1}{2} J_{0x} \dot{\varphi}_0^2 + \frac{1}{2} J_{0y} \dot{\psi}_0^2 + \frac{1}{2} J_{pxf} (\dot{\varphi}_0 + \dot{\varphi}_1)^2 + \\
&+ \frac{1}{2} J_{pxf} (\dot{\varphi}_0 + \dot{\varphi}_2)^2 + \frac{1}{2} J_{pyb} (\dot{\psi}_0 + \dot{\psi}_3)^2 + \frac{1}{2} J_{pyb} (\dot{\psi}_0 + \dot{\psi}_4)^2 + \\
&+ 2(\frac{1}{2} J_{pyf} \dot{\psi}_0^2) + 2(\frac{1}{2} J_{pxb} \dot{\varphi}_0^2) + \frac{1}{2} m_p (\dot{z}_0 - (L_{mpf} + b_f) \dot{\varphi}_0 + \\
&+ L_f \dot{\psi}_0 - L_{mpf} \dot{\varphi}_1)^2 + \frac{1}{2} m_p (\dot{z}_0 + (L_{mpf} + b_f) \dot{\varphi}_0 + L_f \dot{\psi}_0 + \\
&+ L_{mpf} \dot{\varphi}_2)^2 + \frac{1}{2} m_p (\dot{z}_0 - b_b \dot{\varphi}_0 - (L_{mpb} + L_b) \dot{\psi}_0 - L_{mpb} \dot{\psi}_3)^2 + \\
&+ \frac{1}{2} m_p (\dot{z}_0 + b_b \dot{\varphi}_0 - (L_{mpb} + L_b) \dot{\psi}_0 - L_{mpb} \dot{\psi}_4)^2 + \\
&+ \frac{1}{2} J_d (\dot{\varphi}_0 + \dot{\varphi}_d)^2 + \frac{1}{2} J_d (\dot{\psi}_0 + \dot{\psi}_d)^2 + \frac{1}{2} m_d (\dot{z}_0 + L_d \dot{\psi}_0 - \dot{z}_d)^2
\end{aligned}$$

The potential energy of the system is:

$$\begin{aligned}
\Pi &= \frac{1}{2} c_{rf} (L_{cf} \varphi_1)^2 + \frac{1}{2} c_{rf} (-L_{cf} \varphi_2)^2 + \frac{1}{2} c_{rb} (L_{cb} \psi_3)^2 + \frac{1}{2} c_{rb} (L_{cb} \psi_4)^2 + \\
&+ \frac{1}{2} c_{gz} (z_0 - (b_f + b_{kf}) \varphi_0 + L_f \psi_0 - b_{kf} \varphi_1)^2 + \\
&+ \frac{1}{2} c_{gz} (z_0 + (b_f + b_{kf}) \varphi_0 + L_f \psi_0 + b_{kf} \varphi_2)^2 + \\
&+ \frac{1}{2} c_{gz} (z_0 - (b_b + b_{kb}) \varphi_0 - (L_b + L_{kb}) \psi_0 - L_{kb} \psi_3{}^2 + \\
&+ \frac{1}{2} c_{gz} (z_0 + (b_b + b_{kb}) \varphi_0 - (L_b + L_{kb}) \psi_0 - L_{kb} \psi_4)^2 + \\
&+ \frac{1}{2} c_{sf} (-L_{sf} \varphi_1 - L_{sf} \varphi_2)^2 + \frac{1}{2} c_{sb} (-L_{sb} \psi_3 + L_{sb} \psi_4)^2 + \\
&+ \frac{1}{2} c_d (b_d \varphi_d - l_d \psi_d + z_d)^2 + \frac{1}{2} c_d (-b_d \varphi_d - l_d \psi_d + z_d)^2 + \\
&+ \frac{1}{2} c_d (b_d \varphi_d + l_d \psi_d + z_d)^2 + \frac{1}{2} c_d (-b_d \varphi_d + l_d \psi_d + z_d)^2
\end{aligned}$$

The Rayleigh’s function is:

$$\begin{aligned}
R &= \frac{1}{2} \beta_{rf} (L_{cf} \dot{\varphi}_1)^2 + \frac{1}{2} \beta_{rf} (-L_{cf} \dot{\varphi}_2)^2 + \frac{1}{2} \beta_{rb} (L_{cb} \dot{\psi}_3)^2 \\
&+ \frac{1}{2} \beta_{rb} (L_{cb} \dot{\psi}_4)^2 + \frac{1}{2} \beta_{gz} (\dot{z}_0 - (b_f + b_{kf}) \dot{\varphi}_0 + L_f \dot{\psi}_0 - b_{kf} \dot{\varphi}_1)^2 + \\
&+ \frac{1}{2} \beta_{gz} (\dot{z}_0 + (b_f + b_{kf}) \dot{\varphi}_0 + L_f \dot{\psi}_0 + b_{kf} \dot{\varphi}_2)^2 + \\
&+ \frac{1}{2} \beta_{gz} (\dot{z}_0 - (b_b + b_{kb}) \dot{\varphi}_0 - (L_b + L_{kb}) \dot{\psi}_0 - L_{kb} \dot{\psi}_3{}^2 + \\
&+ \frac{1}{2} \beta_{gz} (\dot{z}_0 + (b_b + b_{kb}) \dot{\varphi}_0 - (L_b + L_{kb}) \dot{\psi}_0 - L_{kb} \dot{\psi}_4)^2 + \\
&+ \frac{1}{2} \beta_{sf} (-L_{sf} \dot{\varphi}_1 - L_{sf} \dot{\varphi}_2)^2 + \frac{1}{2} \beta_{sb} (-L_{sb} \dot{\psi}_3 + L_{sb} \dot{\psi}_4)^2 + \\
&+ \frac{1}{2} \beta_d (b_d \dot{\varphi}_d - l_d \dot{\psi}_d + \dot{z}_d)^2 + \frac{1}{2} \beta_d (-b_d \dot{\varphi}_d - l_d \dot{\psi}_d + \dot{z}_d)^2 + \\
&+ \frac{1}{2} \beta_d (b_d \dot{\varphi}_d + l_d \dot{\psi}_d + \dot{z}_d)^2 + \frac{1}{2} \beta_d (-b_d \dot{\varphi}_d + l_d \dot{\psi}_d + \dot{z}_d)^2
\end{aligned}$$

After applying Lagrange's equation of 2nd kind:

$$\frac{d}{dt}\left(\frac{\partial \mathrm{T}}{\partial \dot{q}}\right) - \left(\frac{\partial \mathrm{T}}{\partial q}\right) = -\left(\frac{\partial \Pi}{\partial q}\right) - \left(\frac{\partial R}{\partial \dot{q}}\right),$$

for equations describing the laws of motion of the system of Figure 1 is valid:

$$[\mathrm{M}]\ddot{q} + [\mathrm{B}]\dot{q} + [C]q = [0]$$

- [M] is the matrix of inertia that is symmetrical with the main diagonal with dimension 10x10 and she has the type of Table. 2
- [C] is the matrix of elasticity, which is also symmetric and has

10x10 dimension (Table 3)

- [B] is the matrix of dissipative forces, showing the influence of dampers - with a symmetrical dimension 10x10 (Table 4):

Generalized coordinates and their derivatives are:

$$\{q\}=\begin{bmatrix} z_0 \\ \varphi_0 \\ \psi_0 \\ \varphi_1 \\ \varphi_2 \\ \psi_3 \\ \psi_4 \\ z_d \\ \varphi_d \\ \psi_d \end{bmatrix} \quad \{\dot{q}\}=\begin{bmatrix} \dot{z}_0 \\ \dot{\varphi}_0 \\ \dot{\psi}_0 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\psi}_3 \\ \dot{\psi}_4 \\ \dot{z}_d \\ \dot{\varphi}_d \\ \dot{\psi}_d \end{bmatrix} \quad \{\ddot{q}\}=\begin{bmatrix} \ddot{z}_0 \\ \ddot{\varphi}_0 \\ \ddot{\psi}_0 \\ \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\psi}_3 \\ \ddot{\psi}_4 \\ \ddot{z}_d \\ \ddot{\varphi}_d \\ \ddot{\psi}_d \end{bmatrix}$$

To obtain natural frequencies of the system, equations are presented in the normal Cauchy form:

$$y + Ly = 0$$

Where L is:

$$L=\begin{bmatrix} M^{-1}B & M^{-1}C \\ I & O \end{bmatrix}$$

Table 2:

$m_0+4m_p+m_d$	0	$2m_pL_f-2m_p(L_{mpb}+L_b)+m_dL_d$	$-m_pL_{mpf}$	m_pL_{mpf}	$-m_pL_{mpb}$	$-m_pL_{mpb}$	$-m_d$	0	0
0	$J_{0x}+2J_{pxf}+2J_{pxb}+2m_pb_b^2+2m_p(L_{mpf}+b_f)^2+J_d$	0	$J_{pxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$J_{pxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$m_pb_bL_{mpb}$	$-m_pb_bL_{mpb}$	0	J_d	0
$2m_pL_f-2m_p(L_{mpb}+L_b)+m_dL_d$	0	$J_{0y}+2J_{pyb}+2J_{pyf}+2m_pL_f^2+2m_p(L_{mpb}+L_b)^2+J_d+m_dL_d^2$	$-m_pL_fL_{mpf}$	$m_pL_fL_{mpf}$	$J_{pyb}+m_pL_{mpb}(L_{mpb}+L_b)$	$J_{pyb}+m_pL_{mpb}(L_{mpb}+L_b)$	$-m_dL_d$	0	J_d
$-m_pL_{mpf}$	$J_{pxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$-m_pL_fL_{mpf}$	$J_{pxf}+m_pL_{mpf}^2$	0	0	0	0	0	0
m_pL_{mpf}	$J_{pxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$m_pL_fL_{mpf}$	0	$J_{pxf}+m_pL_{mpf}^2$	0	0	0	0	0
$-m_pL_{mpb}$	$m_pb_bL_{mpb}$	$J_{pyb}+m_pL_{mpb}(L_{mpb}+L_b)$	0	0	$J_{pyb}+m_pL_{mpb}^2$	0	0	0	0
$-m_pL_{mpb}$	$-m_pb_bL_{mpb}$	$J_{pyb}+m_pL_{mpb}(L_{mpb}+L_b)$	0	0	0	$J_{pyb}+m_pL_{mpb}^2$	0	0	0
$-m_d$	0	$-m_dL_d$	0	0	0	0	m_d	0	0
0	J_d	0	0	0	0	0	0	J_d	0
0	0	J_d	0	0	0	0	0	0	J_d

Table 3:

$4c_{gz}$	0	$2c_{gz}L_f-2c_{gz}(L_b-L_{kb})$	$-c_{gz}b_{kf}$	$c_{gz}b_{kf}$	$-c_{gz}L_{kb}$	$-c_{gz}L_{kb}$	0	0	0
0	$2c_{gz}(b_f+b_{kf})^2+2c_{gz}(b_b+b_{kb})^2$	0	$c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gz}L_{kb}(b_b+b_{kb})$	$-c_{gz}L_{kb}(b_b+b_{kb})$	0	0	0
$2c_{gz}L_f-2c_{gz}(L_b+L_{kb})$	0	$2c_{gz}L_f^2+2c_{gz}(L_b+L_{kb})^2$	$-c_{gz}b_{kf}L_f$	$c_{gz}b_{kf}L_f$	$c_{gz}L_{kb}(L_b+L_{kb})$	$c_{gz}L_{kb}(L_b+L_{kb})$	0	0	0
$-c_{gz}b_{kf}$	$c_{gz}b_{kf}(b_f+b_{kf})$	$-c_{gz}b_{kf}L_f$	$c_{rf}L_{cf}^2+c_{gz}b_{kf}^2+c_{sf}L_{sf}^2$	$c_{sf}L_{sf}^2$	0	0	0	0	0
$c_{gz}b_{kf}$	$c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gz}b_{kf}L_f$	$c_{sf}L_{sf}^2$	$c_{rf}L_{cf}^2+c_{gz}b_{kf}^2+c_{sf}L_{sf}^2$	0	0	0	0	0
$-c_{gz}L_{kb}$	$c_{gz}L_{kb}(b_b+b_{kb})$	$c_{gz}L_{kb}(L_b+L_{kb})$	0	0	$c_{rb}L_{cb}^2+c_{gz}L_{kb}^2+c_{sb}L_{sb}^2$	$-c_{sb}L_{sb}^2$	0	0	0
$-c_{gz}L_{kb}$	$-c_{gz}L_{kb}(b_b+b_{kb})$	$c_{gz}L_{kb}(L_b+L_{kb})$	0	0	$-c_{sb}L_{sb}^2$	$c_{rb}L_{cb}^2+c_{gz}L_{kb}^2+c_{sb}L_{sb}^2$	0	0	0
0	0	0	0	0	0	0	$4c_d$	0	0
0	0	0	0	0	0	0	0	$4c_db_d^2$	0
0	0	0	0	0	0	0	0	0	$4c_dl_d^2$

Table 4:

$4\beta_{gz}$	0	$2\beta_{gz}L_f-2B_{gz}(L_b-L_{kb})$	$-\beta_{gz}b_{kf}$	$\beta_{gz}b_{kf}$	$-\beta_{gz}L_{kb}$	$-\beta_{gz}L_{kb}$	0	0	0
0	$2\beta_{gz}(b_f+b_{kf})^2+2B_{gz}(b_b+b_{kb})^2$	0	$\beta_{gz}b_{kf}(b_f+b_{kf})$	$\beta_{gz}b_{kf}(b_f+b_{kf})$	$\beta_{gz}L_{kb}(b_b+b_{kb})$	$-\beta_{gz}L_{kb}(b_b+b_{kb})$	0	0	0

$2\beta_{gz}L_f - 2\beta_{gz}(L_b+L_{kb})$	0	$2\beta_{gz}L_f^2+2\beta_{gz}(L_b+L_{kb})^2$	$-\beta_{gz}b_{kf}L_f$	$\beta_{gz}b_{kf}L_f$	$\beta_{gz}L_{kb}(L_b+L_{kb})$	$\beta_{gz}L_{kb}(L_b+L_{kb})$	0	0	0
$-\beta_{gz}b_{kf}$	$\beta_{gz}b_{kf}(b_f+b_{kf})$	$-\beta_{gz}b_{kf}L_f$	$\beta_{rf}L_{cf}^2+\beta_{gz}b_{kf}^2+\beta_{sf}L_{sf}^2$	$\beta_{sf}L_{sf}^2$	0	0	0	0	0
$\beta_{gz}b_{kf}$	$\beta_{gz}b_{kf}(b_f+b_{kf})$	$\beta_{gz}b_{kf}L_f$	$\beta_{sf}L_{sf}^2$	$\beta_{rf}L_{cf}^2+\beta_{gz}b_{kf}^2+\beta_{sf}L_{sf}^2$	0	0	0	0	0
$-\beta_{gz}L_{kb}$	$\beta_{gz}L_{kb}(b_b+b_{kb})$	$\beta_{gz}L_{kb}(L_b+L_{kb})$	0	0	$\beta_{rb}L_{cb}^2+\beta_{gz}L_{kb}^2+\beta_{sb}L_{sb}^2$	$-\beta_{sb}L_{sb}^2$	0	0	0
$-\beta_{gz}L_{kb}$	$-\beta_{gz}L_{kb}(b_b+b_{kb})$	$\beta_{gz}L_{kb}(L_b+L_{kb})$	0	0	$-\beta_{sb}L_{sb}^2$	$\beta_{rb}L_{cb}^2+\beta_{gz}L_{kb}^2+\beta_{sb}L_{sb}^2$	0	0	0
0	0	0	0	0	0	0	$4\beta_d$	0	0
0	0	0	0	0	0	0	0	$4\beta_d b_d^2$	0
0	0	0	0	0	0	0	0	0	$4\beta_d l_d^2$

3. Numerical investigations

MATLAB is powerful control system simulation software and widely used in engineering filed.

The main parameters and their numerical values (Table 5) are not measured by the authors and are taken from cited literary sources:

Table 5:

№	Parameter	Symbol	Value
1.	Suspended masses	m_0	1100 kg
2.	Unsuspended masses	m_p	30 kg
3	Mass of the power unit	m_d	300 kg
4.	Moment of inertia of the suspended masses around longitudinal axis (x-axis)	J_{0x}	550 kg.m^2
5.	Moment of inertia of the suspended masses around transverse axis (y-axis)	J_{0y}	2000 kg.m^2
6.	Moment of inertia of the unsuspended masses on the front axle around x-axis	J_{pxf}	5 kg.m^2
7.	Moment of inertia of the unsuspended masses on the rear axle around x-axis	J_{pxb}	2 kg.m^2
8.	Moment of inertia of the unsuspended masses on the front axle around y-axis	J_{pyf}	2 kg.m^2
9.	Moment of inertia of the unsuspended masses on the rear axle around y-axis	J_{pyb}	5 kg.m^2
10	Moment of inertia of the power unit around Ox and O_y	J_d	100 kg.m^2
12.	Vertical co-ordinate of the center of gravity of the unsuspended masses in relation to joint of the arms	H	0,4 m
13.	Horizontal co-ordinate of the center of gravity of the unsuspended masses in relation to joint of the front arms	b_f	0,4 m
14.	Horizontal co-ordinate of the center of gravity of the unsuspended masses in relation to joint of the rear arms	b_b	0,6 m
15.	Distance from the center of gravity to the front axle	L_f	1,1 m
16.	Distance from the center of gravity to the rear axle	L_b	1,5 m
17.	Vertical co-ordinate of the center of gravity of the power unit	H_d	0,2 m
18.	Length of the front(f) and the rear(f) arm	b_k	0,42 m
19.	Distance from the contact point of the rear wheel to joint of the arm	b_{kb}	0,2 m
20.	Distance from the center of gravity of the front(f) and the rear(b) arm to the respective joint	L_{mp}	0,4 m
21.	Distance from fixing point of the front(f) and the rear(b) main elastic element to the respective joint	L_c	0,3 m
22.	Distance from fixing point of the front(f) and the rear(b) anti-roll bar to the respective joint	L_s	0,28 m
23.	Distance from the center of gravity of the power unit to the center of gravity of the car body.	L_d	1,2 m
24.	Distance from the center of gravity of the power unit to the attachment points of his mountings along the axes Ox and Oy.	l_d и b_d	0,6 m
25.	Radius of the front(f) and the rear(b) wheels	R_k	0,26 m
26.	Stiffness coefficient of the main elastic elements of the front axle	c_{rf}	25000 N/m
27.	Stiffness coefficient of the main elastic elements of the rear axle	c_{rb}	25000 N/m
28.	Stiffness coefficient of the tyre	c_{gz}	125000 N/m
29.	Stiffness coefficient of the anti-roll bars of the front(f) and the rear(b) axle	c_s	20000 N/m
30.	Stiffness coefficient of the mountings	c_d	300000 N/m
31.	Damping coefficient of the front(f) and the rear(b) shock absorbers	β_r	1900 N.s/m
32.	Damping coefficient of the mountings	β_d	500 N.s/m

The simulation results for the natural frequencies of the systeme are:

-0.7946 +12.2936i
-0.7946 -12.2936i
-0.5854 +10.5659i
-0.5854 -10.5659i
-0.2488 + 1.0363i
-0.2488 - 1.0363i
-0.1206 + 0.7344i
-0.1206 - 0.7344i
-1.3950 + 7.8194i
-1.3950 - 7.8194i
-1.4713 + 7.7920i

-1.4713 - 7.7920i
-0.6867 +11.3617i
-0.6867 -11.3617i
-0.2157 + 1.5422i
-0.2157 - 1.5422i
-1.6600 + 8.2602i
-1.6600 - 8.2602i
-1.5831 + 8.2717i
-1.5831 - 8.2717i

The natural frequency is equal to the imaginary part of eigenvalues of the matrix and the real part relates to damping.

The identification of frequencies is solved by simpler models with fewer degrees of freedom and the results are:

1.0363 Hz – frequency of linear oscillations of the sprung masses on z-axis;

1.5422 Hz – frequency of angular oscillation of the sprung masses around x-axis;

0.7344 Hz – frequency of angular oscillation of the sprung masses around y-axis;

7.7920 и 7.8194 Hz – angular frequency of the front arms;

8.2602 и 8.2717 Hz – angular frequency of the rear arms;

12.2936 Hz - frequency of linear oscillations of the power unit on z-axis;

11.3617 Hz - frequency of angular oscillation of the power unit around x-axis;

10.5659 - frequency of angular oscillation of the power unit around y-axis.

Received natural frequencies of the power unit do not coincide with those of other units of the car, but also within the prescribed limits in the table. 1. Therefore, elastic and damping characteristics of the elements are chosen correctly.

4. Conclusion

The model gives us possibility to obtain natural frequencies to whole system. The results of numerical simulation can be considered more reliable because the model takes into account the effects of power unit.

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