USING TIME WINDOWS FOR MACHINES' FAULT DETECTION BASED ON POWER SPECTRUM ESTIMATION

ИСПОЛЬЗОВАНИЕ ВРЕМЕННЫХ ОКОН ДЛЯ ДЕФЕКТОСКОПИИ МАШИН НА ОСНОВЕ ОЦЕНКИ СПЕКТРА ПЛОТНОСТИ МОЩНОСТИ

Dr. eng. Okarma K., Dr. eng. Mazurek P.
WSTE in Szczecin, Poland

Abstract: In the paper some possibilities of using time windows as a tool for power spectrum estimation and fault detection methods are presented. Choice of proper window function can be on of the crucial elements in vibroacoustics based techniques used for machine's fault detection. Applying time windows with good frequency characteristics' sidelobes' attenuation helps to achieve good separation for signals with similar frequencies even with low amplitude of one of them. The comparison of effects obtained using some popular window functions and also recently derived ones is also presented for the example signal.

KEYWORDS: TIME WINDOWS, POWER SPECTRUM ESTIMATION, FAULT DETECTION, VIBROACOUSTIC MEASUREMENTS

1. Introduction

Diagnostic methods based on vibroacoustics can provide some important information corresponding to the current state of an object and also make possible to perform the fault detection. Acoustic phenomenon can be treated as the representative of the machine's working process and the changes in the acoustic characteristic of that process may testify to improper work or even failure of some parts of the machine. Such vibrations, not necessary in acoustic band only, are also related to some of the most relevant physical processes, which take place during machine's work, such as strains, deformations etc., important for proper working process.

Measuring the vibroacoustic signals there is a possibility of obtaining desired data in normal working mode of the machine during a relatively short period of time, without the necessity of changing the machine's working conditions and affecting any technological processes.

Acoustic signal is characterised by quite big amount of data which can be used as diagnostic information. Acoustic diagnosis can be used then in purpose to determine the state of the machines and also the fact of their (or their element's) inefficiency or waste.

Typical diagnostic signals are based on the derivatives of the signal in the time or frequency domain [1]. Time based characteristics are:

- speed (or counted number of occurrences) of impulse processes,
- number of zero-crossing occurrences (or crossings by the other chosen amplitude),
- maximum amplitude,
- average amplitude,
- maximum RMS value,
- average RMS value,
- area above the average value in the time domain.

Characteristics specific for frequency domain are:

- maximum frequency in the power spectrum,
- median frequency in the power spectrum,
- maximum spectrum's value for specified frequency range,
- average frequency in the spectrum,
- bandwidth of signals above specified amplitude level,
- energy in specified frequency range (e.g. tierce, octave).

Relations between classes and measured acoustic parameters can be analysed as:

- macrogeometrical features (e.g. dimensions, symmetry, shape, relative location of elements etc.),
- microgeometrical features (e.g. roughness),
- dynamic features of vibration source,
- electrical and magnetic features,
- thermal, technological or working features.

2. Spectrum analysis

One of the most useful methods of signal analysis is power spectrum estimation which can be performed using Blackman-Tukey's method utilising the values of the autocorrelation function [2]. Because of the limited time of observation such autocorrelation function is defined in the time interval (-T/2 ; T/2 > where T is the time of observation. It is the cause of the effect of spectral leakage, which can be reduced using appropriate time windows (known also as apodizing functions) treated as the weighting functions in product with the corresponding values of the autocorrelation function.

Choosing the appropriate window function it is also possible to detect some signals which are close to each other in the frequency domain and their amplitudes differ significantly. If the signal which should be detected has much lower amplitude than some other signals with similar frequency it is relevant to use such window that unwanted signals, usually related to proper machine's working state, will be reduced in further analysis. It may be especially relevant in the machine's fault detection where the presence or specified signal's level in some frequency ranges may be related to some elements' waste or failure.

There is a fundamental dependency between time signal u(t) and its spectrum G(f) which can be characterised by the following pair of Fourier transforms

\[ G(f) = F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot \exp(-j \cdot 2 \cdot \pi \cdot f \cdot t) dt \quad (1) \]

and

\[ u(t) = F^{-1}[G(f)] = \int_{-\infty}^{\infty} G(f) \cdot \exp(j \cdot 2 \cdot \pi \cdot f \cdot t) df \quad (2) \]

Due to limited time of observation using infinite boundaries in integration is not possible so the range of integration should be limited what is equivalent to multiplying the signal by the weighting function (time window) with zero values outside the specified range. In the frequency domain it is related to the convolution of the spectrum with the frequency window. It can be then expressed as:

\[ G(f) = F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot w(t) \cdot \exp(-j \cdot 2 \cdot \pi \cdot f \cdot t) dt \quad (3) \]

and
\[
 u(t) = F^{-1}[G(f)] = \int_{-\infty}^{+\infty} G(f) \cdot W(f) \cdot \exp\left(j \cdot 2 \cdot \pi \cdot f \cdot t\right) df
\]  
(4)

where \( u(t) \) is the time window function. For the digital signals the equivalent technique is based on Discrete Fourier Transform (DFT) or much more efficient Fast Fourier Transform (FFT). The most relevant element in acoustic and vibration based analysis is the measurement of amplitude or energetic spectra equivalent to the modulus or squared modulus of the spectra respectively.

3. Window functions

In signal processing applications many different time windows are used which can be characterised by some characteristic parameters. The most often used features are:

- width of the main lobe of the frequency response (WML),
- relative sidelobe attenuation (highest sidelobe level - HSLL),
- sidelobes' decay speed,
- sidelobes to main lobe energy ratio (energetic criterion).

An ideal window function should have the narrowest possible main lobe (high frequency resolution) with minimum level of the sidelobes in purpose to suppress unwanted frequency ranges well. However, the requirements to the sidelobes attenuation and the main lobe's width opposite, so several compromise window functions have been proposed, sometimes with very complicated formulas.

The simplest window function, in fact meaning using no window at all, is the rectangular window and using the triangle (Bartlett) window is the easiest method to weight the signal samples inside the window. Regardless of the fact that rectangular window has the narrowest possible width of the main lobe \((WML=1/T)\), the other parameters of both windows are poor (e.g. only 13 dB sidelobes attenuation for rectangular window).

Some of the most popular windows are cosine-based ones such as von Hann (Hanning) window given as:

\[
 w_{Hn}(\tau) = \left(0.5 + 0.5 \cdot \cos \frac{\pi \tau}{T}\right) \cdot \prod\left(\frac{\tau}{2T}\right)
\]  
(5)

where the observation time is equal to \(2T\) for convenience (usually \(T=1\) is assumed in continuous time analysis) and:

\[
 \prod\left(\frac{\tau}{2T}\right) = \begin{cases} 1 & \text{if } |\tau| \leq T \\ 0 & \text{if } |\tau| > T \end{cases}
\]

is actually the rectangular window limiting the range of window function's definition. Some other popular windows are Hamming and Blackman ones defined as:

\[
 w_{Hm}(\tau) = \left(0.54 + 0.46 \cdot \cos \frac{\pi \tau}{T}\right) \cdot \prod\left(\frac{\tau}{2T}\right)
\]  
(6)

and

\[
 w_{Bl}(\tau) = \left(0.42 + 0.5 \cdot \cos \frac{\pi \tau}{T} + 0.08 \cdot \cos \frac{2\pi \tau}{T}\right) \cdot \prod\left(\frac{\tau}{2T}\right)
\]  
(7)

respectively. However width of the main lobe depends on the number of cosine functions used and is doubled in von Hann and Hamming windows in comparison to the rectangular one (Blackman window has the WML equal to 3). More detailed parameters of such classical windows and some of their modifications can be found in literature e.g. [3, 5].

An interesting alternative for such windows can be using polynomial window functions characterised by low computational complexity and wide possibilities of changing their frequency properties just by choosing appropriate polynomial coefficients.

The definition of such window can be expressed as:

\[
 wp(\tau) = \left(1 + \sum_{n=1}^{N} C_{2n} \left(\frac{\tau}{T}\right)^{2n}\right) \cdot \prod\left(\frac{\tau}{T}\right)
\]  
(9)

assuming only even exponents, where \(2N\) denotes the order of polynomial window.

Obtaining fast decaying sidelobes [4] requires forcing the specified number of derivatives to zero in the boundary points of the observation's time interval at the cost of high level of the first sidelobe. In some detection applications such poor sidelobes attenuation may be insufficient. However, resigning from forcing some of the derivatives to zero the optimisation of the highest sidelobe level is also possible [6] leading to the family of windows with good sidelobe attenuation with slightly wider main lobe.

Using polynomial windows it is possible to choose the polynomial coefficients in such way that the suppression of the undesired signals is higher than using some classical ones. If the frequencies of the signals related to proper machine's work are known, the polynomial window can be optimised so that those frequencies will be placed exactly between the main lobe and the first sidelobe of the amplitude characteristics or exactly between some of the further sidelobes (Fig. 1).

Optimisation of the polynomial windows is possible assuming the specified width of the main lobe and some additional conditions related to zeroes of its frequency response at specified points related to the frequencies which should be suppressed. Using classical windows such way of optimisation is practically impossible. Assuming 12-th order polynomial windows (with only even exponents) the requirement of the WML equal to 2 as in Hann and Hamming windows expressed as

\[
 W\left(f = \frac{2}{T}\right) = 0
\]  
(10)

is equivalent to the following condition

\[
 C_{12} \cdot \left(\frac{3}{512} - \frac{165}{1024 \pi^2} + \frac{1485}{512 \pi^4} - \frac{31185}{1024 \pi^6} + \frac{155925}{1024 \pi^8} - \frac{467775}{2048 \pi^{10}}\right) \\
 + C_{10} \cdot \left(\frac{5}{256} - \frac{45}{128 \pi^2} + \frac{945}{256 \pi^4} - \frac{4725}{512 \pi^6} + \frac{14175}{1024 \pi^8}\right) \\
 + C_{8} \cdot \left(\frac{1}{16} - \frac{21}{32 \pi^2} + \frac{105}{64 \pi^4} - \frac{315}{128 \pi^6} + \frac{645}{256 \pi^8}\right)
\]  
(11)

Choosing the WML=3 identical as in Blackman window

\[
 W\left(f = \frac{3}{T}\right) = 0
\]  
(12)

the following formula is obtained:

\[
 C_{12} \cdot \left(\frac{3}{512} - \frac{55}{768 \pi^2} + \frac{55}{96 \pi^4} - \frac{385}{144 \pi^6} + \frac{1925}{24 \pi^8} - \frac{1925}{8 \pi^{10}}\right) \\
 + C_{10} \cdot \left(\frac{5}{256} - \frac{35}{32 \pi^2} + \frac{175}{48 \pi^4} - \frac{175}{16 \pi^6} + \frac{175}{8 \pi^8}\right) \\
 + C_{8} \cdot \left(\frac{1}{16} - \frac{7}{24 \pi^2} + \frac{35}{54 \pi^4} - \frac{35}{18 \pi^6} + \frac{81}{8 \pi^8}\right)
\]  
(13)

\[
 C_{6} \cdot \left(\frac{3}{16} - \frac{5}{12 \pi^2} + \frac{5}{18 \pi^4} + \frac{C_{4} \cdot \left(\frac{1}{2} - \frac{1}{3 \pi^2}\right) + C_{2} = 0
\]
Fig. 1. Maximum suppression frequencies illustrated using amplitude characteristics of Hamming window.

Fig. 2. Idea of the suppression of chosen frequencies illustrated using amplitude characteristics of Hamming window (top – spectrum before application of window function, bottom – spectrum after application of the window function).

Fig. 3. Illustration of the weak suppression of chosen frequencies illustrated using amplitude characteristics of Hamming window (top – spectrum before application of window function, bottom – spectrum after application of the window function).

Conditions for the 10-th order windows have been presented in the paper [6]. Applying more conditions with additional limitations forcing the desired decay speed a system of equations is obtained. If the number of equations is smaller than the number of polynomial coefficients additional optimisation using specified criterion is also possible.

As shown in Figures 1 and 2 some windows e.g. Hamming window have almost regularly distributed zero-crossing frequencies in their spectrum. In the case of using polynomial windows the width of each sidelobe can be different. It can be useful especially in the situations when the frequencies of real signals fluctuate, because changing slightly the polynomial coefficients it is possible to suppress chosen signals better without changing the length of the window. The effect which can be obtained for fluctuations of the frequency is illustrated in Figure 3.

Proposed idea of using polynomial windows with variable characteristic can be joined with some optimisation or tracking techniques which should operate in real-time. Additional computational complexity related to the calculation of the window function's values or its spectrum is not high.

In purpose to compare the influence of using various windows the test signal with known spectral characteristics should be used. In the opposite case for the signal with unknown spectral components the influence of the windows' properties may be hard to determine.

In this paper as the test signal two sinusoidal waves with different amplitudes are chosen but in real situation the spectrum of the signal is usually more complicated.
In purpose to illustrate the wide possibilities of changing the polynomial windows’ frequency properties, some of the windows’ characteristics obtained during experiments with various signals are presented in Figure 5.

4. Conclusions

Polynomial windows are an interesting alternative for many commonly used time windows. One of the most interesting features are wide possibilities of changing their properties just by choosing appropriate polynomial’s order and coefficients. As shown in Figures 4 and 5, changing the polynomial coefficients influences the time shape of the window function as well as the amplitude characteristics. Zero-crossing frequencies differ significantly so it is possible to achieve polynomial windows with practically any desired zero-crossing points in their frequency responses.

5. Bibliography